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SOFT COMPUTING WITH QUADRATIC SPLINE FOR OPTIMIZED CURVES OF SURFACE/SURFACE INTERSECTIONS

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Abstract

The intersection of two surfaces, approximating curves, has been seen one of problematic issue in geometrics, computer graphics & engineering. With the rising interest in computing design field, surface intersection specifically turned into focal point of several studies. Its significant application in these fields makes it yet fascinating topic for research. This paper targets to lessening time and error needs for computation cycle of surface intersection. For this purpose, a robust algorithm for approximating parametric-parametric, explicit-explicit, explicit-implicit surface intersection curves is proposed in this study. The proposed process begins with the identification of "turning & boundary" points from an array of surface intersection points, and then uses an iterative optimization technique called Genetic Algorithm (GA) with a quadratic spline to approximate these locations. It has been used to determine the best fit curve by determining the optimum shape parameter values specified in the quadratic spline description. The suggested approach is advantageous since it does not include any unnecessary data points for the intended objectives. Experiments show that this strategy is successful and a superior option for locating solutions to surface intersection issues with the least amount of error.

Keywords: Surface intersection, characteristics points, genetic algorithm, Approximation, Quadratic spline.

1. INTRODUCTION

Surface intersection addresses various problems in numerous engineering applications' including industrial, aeronautical, and engineering drawing. National Aeronautics and Space Administration (NASA) uses several intersection techniques to develop, test and improve models (including technical drawings). Engineers are still working on discover large-scale auto-stimulate things as in aerospace engineering, where complicated mathematical computational geometry is utilized to create aircraft and spacecraft. Parametric, explicit and implicit surfaces along with pyramids, prisms and cones are consumed for intersection in all these branches of engineering. Parametric surfaces are popular for their smoothness and ability to meet basic design needs. While constructing

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surface intersection curves, many complications and poor results arise [9] due to large scale of numerical calculation.

Surface intersections, both explicit and parametric, are most typically employed in these applications since they are a little easier to manage. It's not easy to implicitize at least one of the surfaces when both are supplied parametrically. The fundamental idea of [2] is to collect all of the piece-wise intersection patches, which comprise all of the closed loop's border and turning points. The approach uses a tracing technique to save each component's beginning location and then find them using the marching method. Same methodology has been extended in [3], [4] but they have different critical issues like usage of higher derivatives for domain (u, v) and authors chose complex Hausdroff function for error, not helpful in finding appropriate outcomes. The technique in [6] is not useful for surface intersection since it requires the rearranging of intersecting locations, which is a time-consuming procedure. In addition, numerous iterative processes are used in [8] to find locations on Rational Polynomial Parametric surface intersection curves with high error fluctuation. Accuracy, robustness, and efficiency are the primary objectives of the surface intersection algorithm. This paper introduces a robust surface intersection method that addresses all obstacles described above and gives good results with least error.

The Genetic Algorithm (GA) is a collection of techniques designed to demonstrate and enable the creation of outcomes for real-world issues that are difficult to represent theoretically. Its major goal is to reduce uncertainty, imprecision, and robustness. Soft computing, in reality, functions similarly to the human mind [10] and the outcomes of difficult issues. Numerous optimization techniques are in literature for example Neural Networks, Genetic Algorithm, Fuzzy Logics, Simulated Annealing [11] and Evolutionary computing. The heuristic technique in [1] is to deal optimization problems by using ruled parametric surfaces.

The strategy of Surface/Surface intersection (parametric-parametric, explicit-implicit, and explicit-explicit) is accomplished in this study, as well as the extraction of boundary and turning points using the proposed approach. For good estimated results, a quadratic spline and a Genetic Algorithm are implemented.

1.1 Strength and Limitations

The proposed strategy includes implicit surfaces which are relatively difficult to handle during programming as compared to explicit and parametric surfaces therefore implicit-implicit surface intersection case has not been discussed. It consumes time in every step for showing results which makes the technique's scale unsuccessful; while other cases have adaptability to determine all potential challenges during each phase of iteration. This research technique is not limited to only the cases discussed in this paper but it can also be applied on other surface intersection combinations like implicit-parametric, explicit-parametric, implicit algebraic-rational polynomial parametric etc. Most influential advantage of this approach is; it gives good results in lesser iterations & characteristics points with minimized error (by Least Square's Error technique). Precision, robustness and effectiveness is the main goal of this research which has more impact in this scientific world on researchers.

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The paper is summarized as: section 2 includes phases of surface intersection. Section 3 discusses quadratic spline for curve fitting. Genetic Algorithm along with optimal quadratic spline. Demonstrations, comparison and conclusion are given in section 4 , 5 and section 6.

2. PHASES OF SURFACE INTERSECTION BY QUADRATIC SPLINE

This section describes the whole methodology of surface intersection with quadratic spline.

- Various types of surface intersection are explored in the literature [2], [7], [8], all of which have significant problems during intersection and do not provide adequate results. The first step is to locate the intersection of two surfaces. This thesis examines the intersection of parametric-parametric, implicit-explicit, and explicit-explicit surfaces. These combinations are difficult to solve, but the suggested approach makes it feasible by presenting a single algorithm that can handle all types of problems and produce satisfactory results. Second phase includes the calculation of turning & boundary points.
- Third phase describes that calculated points in second phase are approximated through quadratic spline. Genetic algorithm with quadratic spline is also very helpful in optimal curve fitting.

3. EXTRACTION OF TURNING & BOUNDARY POINTS

In this step user has to extract sequence of points of intersections of two input surfaces.

Input surfaces may include two "explicit" surfaces S_1 & S_2 in R^3 , represented by:

$$G_1: \{S_1 = A_1(x_1, x_2): a_1 \le x_1 \le a_2, b_1 \le x_2 \le b_2\}$$

$$G_2: \{S_2 = A_2(x_1, x_2): a_1 \le x_1 \le a_2, b_1 \le x_2 \le b_2\}$$

or next combination would be one "implicit" surface C_1 & other is "explicit" surface C_2, denoted as:

$$C_1: \{B_1(x_1, x_2, x_3) = 0: a_1 \le x_1 \le a_2, b_1 \le x_2 \le b_2, c_1 \le x_3 \le c_2\}$$

$$C_2: \{Z_1 = B_2(x_1, x_2): a_1 \le x_1 \le a_2, b_1 \le x_2 \le b_2\}$$

or representation of two "parametric" surfaces R_1 & R_2 is defined as:

$$E_1 = \{R_1(x(s_1, t_1), y(s_1, t_1), z(s_1, t_1)): a_1 \le s_1 \le a_2, b_1 \le t_1 \le b_2\}$$

$$E_2 = \{R_2(x(r_1, w_1), y(r_1, w_1), z(r_1, w_1)): a_1 \le r_1 \le a_2, b_1 \le w_1 \le b_2\}$$

$\forall\ a_1, a_2, b_1, b_2, c_1, c_2 \in R$

For intersection sequence points of above blend of surfaces following action is required

$$\begin{cases} S_1 - S_2 = 0 \\ Z_1 - B_1(x_1, x_2, x_3) = 0 \\ R_1 - R_2 = 0 \end{cases} \qquad (1)$$

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Equation (1) comprises of nonempty intersection which includes sequence points.

3.1 Boundary points

Following conditions are utilized for determination of boundary points:

$x_1 = a_1, x_1 = a_2, x_2 = b_1, x_2 = b_2, x_3 = c_1, x_3 = c_2, s_1 = a_1, s_1 = a_2, t_1 = b_1, t_1 = b_2, r_1 = a_1, r_1 = a_2, w_1 = b_1, w_1 = b_2$ in equation (1)

3.2 Turning points

Those points where intersected curve changes its direction or curvature are called turning points. These points can be found out by calculating "inflection points" of eq. (1) or where slope of intersected curve (1) is 1, 0, and -1.

3.3 Quadratic Spline Function

Curve fitting for detected points, quadratic spline [5] is used. Suppose control points of quadratic spline are F_i , Z_i , F_{i+1} $i \in Z$ with equivalent tangents D_i , D_{i+1} at corner/interpolating points. It contains two shape parameters r_i and s_i of ith segment, whose values affect the shape of curve. Quadratic spline embraces two conic segments. First conic moves through corner points F_i and Z_i while second conic moves through corner points Z_i and F_{i+1}. Conic 1 is defined in equation (2), satisfies convex hull property for F_i , V_i, Z_i .

$$P_i(t) = F_i(1-\theta)^2 + 2V_i(1-\theta)\theta + Z_i\theta^2 \qquad (2)$$

Conic 2 is given in equation (3), fulfills convex hull property for Z_i, W_i, F_{i+1} .

$$P_i^*(t) = Z_i(1-\theta^*)^2 + 2W_i(1-\theta^*)\theta^* + F_{i+1}{\theta^*}^2 \qquad (3)$$

Following values of V_i , W_i , Z_i must satisfy by equation (2) and (3)

$$V_i = F_i + \frac{1}{2} r_i h_i D_i$$

$$W_i = F_{i+1} - \frac{1}{2} s_i h_i D_{i+1}$$

$$Z_i = \frac{V_i + W_i}{2}$$

3.4 Genetic Algorithm (GA)

Genetic Algorithms (GAs) are evolutionary algorithms that use the natural evolution process to accomplish tasks. GA is based on the arrangement of bit strings called chromosomes. To get high-quality output chromosomes, it employs three operators (selection, crossover, and mutation).

GA begins with the selection of a population of chromosomes at random. Because population size is so important for convergence, it should be as large as possible (maximizes or minimizes the cost function). In GA, a collection of chromosomes is used as input, and then a cost function is used to these variables to generate output. Crossover produces two offspring from two parent chromosomes, whereas mutation introduces a

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new chromosome that was not present in the original population. Figure (1) demonstrates how chromosomes family is modified by crossover and mutation operator.

Crossover point

Figure 1: Implementation of Crossover & Mutation operator

Main characteristic of GA, in this paper, is to get optimal solutions of curve fitting. It aids in determining the desired values of shape parameters r_i , s_i specified in explanation of quadratic spline. GA also reduces error; occur during optimization process, which leads to get best approximated results. This iterative method works very efficiently for making this technique more influential. Table (1) shows whose parameters which are utilized during optimization procedure of GA.

Table 1: Parameters for GA

Sr. No.	Name	Values
1	Population size	25-30
2	Genome length	15
3	Selection rate	0.5
4	Mutation rate	0.01

3.5 Optimal Quadratic Function

The main purpose of this proposed scheme is to attain best curve by utilizing optimized values. It could only be possible when least error occurs between actual intersection points and corresponding points on approximated curve. Hence, appropriate optimal shape parameter values are required for minimum squared sum error.

For given set of data points $P_{i,j} = (x_{ij}, y_{ij})$ for $i = 1,2,\dots,n$, $j = 1,2,\dots,m_i$ the minimum squared sum of distance amid $P_{i,j}'s$ and its parametric values$P(t_j)$'s on curve can be specified as:

$$S_i = \sum_{j=1}^{m_i}\left[P_i(u_{i,j}) - P_{i,j}\right]^2 \; i = 1,2,\dots,n$$

where $u's$ are parameterized by chord length, utilized for both conics.

Conic 1:

For conic 1, given in equation (2), sum squared is defined as:

$$S_i = \sum_{j=1}^{m_i}\left[P_i(u_{i,j}) - P_{i,j}\right]^2 \; i = 1,2,\dots,n$$

Conic 2:

Correspondingly, conic 2 is described in equation (3), the squared sum can be considered as:

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$S_i^* = \sum_{j=1}^{m_i} \left[P_i^*(u_{i,j}) - P_{i,j}\right]^2 \; i = 1,2, \dots, n$

The main feature of this technique which proves it better than others is minimal error. It means, in approximated curve fit, the intersection points are in optimal manner, because error is considered as so small rather almost negligible, which are shown in Table 2, 3 & 4.

3.6 Algorithm

Step 1: Insert the data.
Step 2: Discover surface intersections and determine all required boundary and turning points.
Step 3: Suitable values of shape parameters s_i , r_i should be selected by GA.
Step 4: Fit quadratic spline with GA to detected boundary and turning points. If the curve obtained is optimal i.e. required threshold for error is achieved, then go to step 5, otherwise go to step 3 and repeat until optimal curve is attained.
Step 5: Stop

4. DEMONSTRATIONS

Proposed algorithm in section 5 has been implemented on explicit-explicit, explicit-implicit parametric-parametric surfaces. Examples and their discussion have clearly shown that how optimal curve is accomplished.

Example 1: Two explicit surfaces are given by: (see figs 1(a-h))

$Es_1: z = -2x^3 - y^3$

$Es_2: z = 2y^3 - 4x^3 + 1$

Figure 2(a) and 2(b) displays explicit surfaces whereas Figure 2(c) shows the intersection in 3-dimension while 2-dimensional view with characteristics points are given in Figure 2(d). Characteristics points (boundary & turning points) without surfaces are demonstrated in Figure 2(e). Figure 2(f) & 2(g) illustrates the curve fitted by quadratic spline with implementation of 1^{st} and 2^{nd} iteration of GA. Figure 2(h) at 20^{th} iteration shows final optimal curve.

Example 2: Implicit and explicit surfaces are given by: (see figs 2(a-h))

$Is_1: \frac{x^2}{1} + \frac{y^2}{4} + \frac{z^2}{9} = 1$

$Is_2: z = x^2 + 2y^2$

Figure 3(a) and 3(b) portrays implicit and explicit surfaces. Figure 3(c) demonstrates the intersection in 3-dimension while 2-dimensional view with characteristics points are given in Figure 3(d). Characteristics points (boundary & turning points) without surfaces are displayed in Figure 3(e). Figure 3(f) & 3(g) illustrates the curve fitted by quadratic spline with implementation of 1^{st} and 2^{nd} iteration of GA. Figure 3(h) at 24^{th} iteration exhibits final optimal curve.

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Example 3: Two parametric surfaces are given by: (see figs 3(a-h))

$s_1 = \{(3x, 3y, 18x^2y^2): 0 \le x, y \le 1\}$

$s_2 = \{(3s, 3t, -6s^2 + 6t - 1): 0 \le s, t \le 1\}$

Figure 4(a) & 4(b) depicts portrayal of parametric surfaces. Figure 4(c) displays the intersection in 3-dimension while 2-dimensional view with characteristics points are given in Figure 4(d). Characteristics points (boundary & turning points) without surfaces are demonstrated in Figure 4(e). Figure 4(f) & 4(g) illustrates the curve fitted by quadratic spline with implementation of 1st and 2nd iteration of GA. Figure 4(h) at 30th iteration shows optimal curve.

Some iterative values along with time elapsed for optimal curve of Example 1, 2 & 3 are given in Table 2, 3 & 4.

Table 2: Iterative values and time elapsed to get final curve fit of Example 1

	Shape Parameter Values	**Time elapsed (secs)**	**Error (SSE)**	**Total no. intersection points**	**Characteristics Points**
1st Iteration	r=1.74 , s=1.85	0.080	0.0542	370	(0,-0.6934) (1,0.6934) (-1,-1)
2nd Iteration	r=1.43 , s=1.52	0.072	0.00385		
Optimal curve	r=0.99 , s=0.99	0.052	1.3×10^{-5}		

Table 3: Iterative values and time elapsed to get final curve fit of Example 2

	Shape Parameter Values	**Time Elapsed (secs)**	**Error (SSE)**	**Total no. intersection points**	**Characteristics Points**
1st Iteration	v=1.97, w=1.89	0.053	0.0259	45	(0,1.117,2.489) (0,-1.117.2.489) (0.953,0,0.908) (-0.953,0,0.908)
2nd Iteration	v=1.74, w=1.65	0.046	0.00636		
Optimal curve	v=1.34 , w=1.34	0.034	3.2×10^{-8}		

Table 4: Iterative values and time elapsed to get final curve fit of Example 3

	Shape Parameter Values	**Time Elapsed (secs)**	**Error (SSE)**	**Total no. intersection points**	**Characteristics Points**
1st Iteration	v=1.64, w=1.75	0.032	0.0223	132	(0,0.17) (0.41,0.43) (0.46,1)
2nd Iteration	v=1.44, w=1.53	0.029	0.00087		
Optimal curve	v=1 , w=1	0.075	2.1×10^{-7}		

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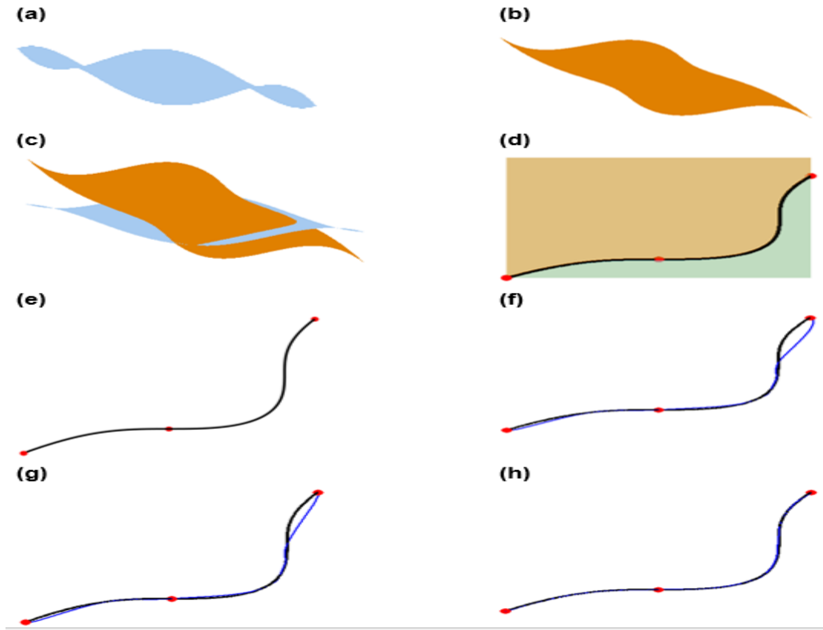

Figure 2: (a) Explicit surface-1 (b) Explicit surface-2 (c) 3-dimensional Surfaces Intersection (d) Characteristics points in xy-view (e) Sequence of points without surfaces (f) Fitted curve by quadratic spline and GA with 1st iteration (g) Fitted curve by quadratic spline and GA with 2nd iteration (h) Final curve attained.

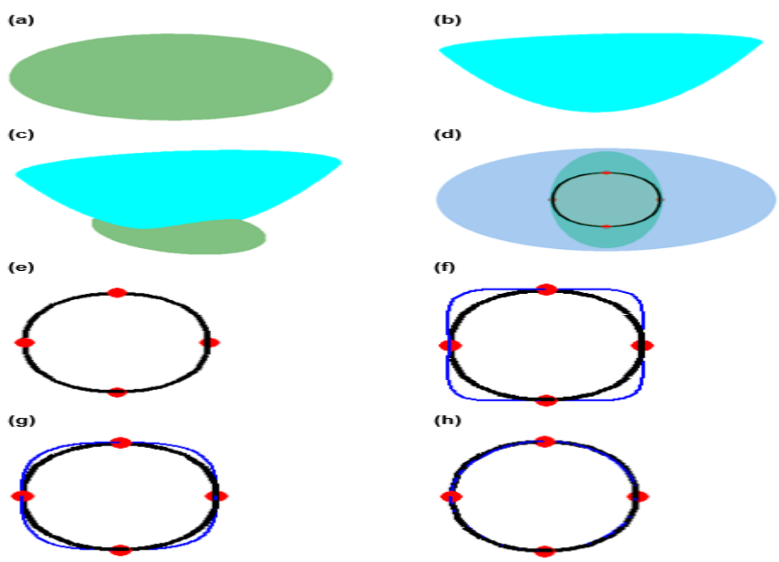

Figure 3: (a) Implicit surface (b) Explicit surface (c) 3-dimensional Surfaces Intersection (d) Characteristics points in xy-view (e) Sequence of points without surfaces (f) Fitted curve by quadratic spline and GA with 1st iteration (g) Fitted curve by quadratic spline and GA with 2nd iteration (h) Final curve attained.

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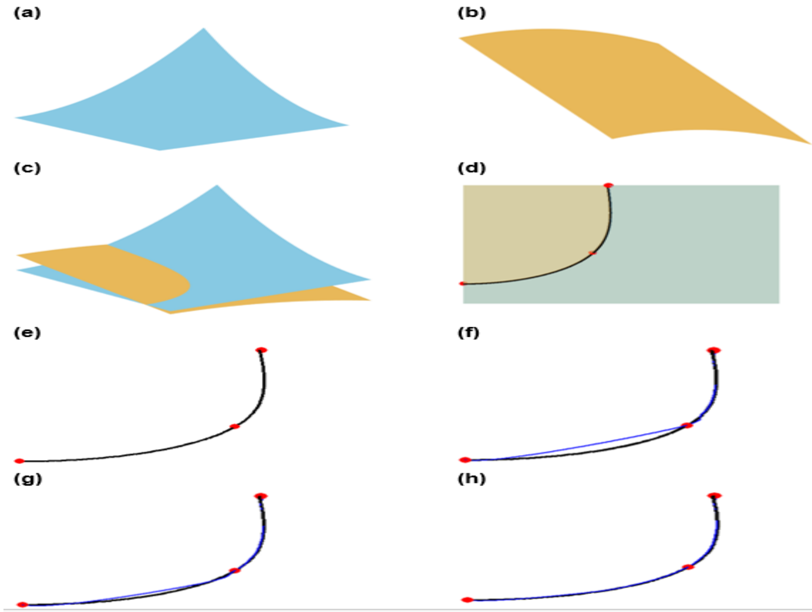

Figure 4: (a) Parametric surface-1 (b) Parametric surface-2 (c) 3-dimensional Surfaces Intersection (d) Characteristics points in xy-view (e) Sequence of points without surfaces (f) Fitted curve by quadratic spline and GA with 1st iteration (g) Fitted curve by quadratic spline and GA with 2nd iteration (h) Final curve attained.

5. COMPARISON

The given table shows the comparison of proposed scheme with existing approaches in literature.

Current schemes	Proposed scheme
In (Hur *et al.,* 2009) Higher derivatives are utilized along with Complicated Hausdroff function for error.	While in this paper simple computational geometry is used behind every calculation and Least squares is utilized for minimal error.
Reordering procedure of intersection points in (Li X. *et al*., 2013 is obviously required time	Whereas here GA iterative procedure makes this technique more influential for outcomes.
Scheme in (Patrikalakis *et al.*, 2013) considered only rational polynomial parametric surface intersection. Also authors used different iterative procedures for surface intersection which results in extreme variation in error.	But as comparison to this scheme Parametric-parametric, implicit-explicit, explicit-explicit surface intersection is discussed and all these types of intersection curves are approximated with one algorithm with less error.

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6. CONCLUSION

This study proposes a surface intersection approach. It may be used to connect parametric-parametric, explicit-implicit, and explicit-explicit surfaces. The recommended method entails finding characteristic points (boundary and turning points) and approximating them using a quadratic spline. The Genetic Algorithm (GA) has also been employed; it aids in reshaping parameter values in order to achieve the needed aim of an ideal curve. The desired curve is attained with the least amount of inaccuracy. The main advantage of this strategy is that it does not require any more points for curve fitting, whereas [12] requires maximum points for approximation.

Most significantly, the approach suggested in this article works well for approximating various types of intersections formed by a variety of surfaces, and it is not limited to a single surface type [8].

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