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LATTICE SPACE IN LQCD FROM QCDLAB2

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Abstract

The computation methods of Lattice Quantum Chromodynamics (LQCD) give possibilities to explore this theory in law energy regimes where nonperturbative methods can be applied. One of the main objectives in lattice QCD calculations is to find the lattice parameter a that it is called lattice scale. The best way to determine this parameter it is the behaviour of quark-anti quark potential. In lattice QCD simulations this potential can come out from calculating before the Wilson loops. In this work, we used dedicated software for lattice QCD simulation, called QCDLAB, version 2.0. In our simulations, to derive the interquark potential we have calculated only planar Wilson loops. As background field configurations of simulations, we have used SU (3) gauge field. These kinds of simulations are repeated for three different values of coupling constant (which means different background field configurations) and are tested increasing the volume of the lattice, specifically for 8^4, 12^4, and 16^4. The calculations are repeated for 100 configurations of gauge fields which are statistically independent. Finally, we find the lattice scale for different lattice volume. If we have lattice space, we can take all physical quantity from quantity measured in lattice unit in physical quantity measured in the physical unit. In our previous work we use old software FERMIQCD for this purpose and now finally with help of author Artan Borici we can use an easy and more effective software such as QCDLAB, version 2.0.

Keywords: Algorithms, Computational Methods, Lattice QCD, Lattice Space, QCDLAB2, Simulations, Wilson Loops

1. INTRODUCTION

Quantum Chromodynamics (QCD) is the theory that describes the properties of the strong interactions between particles in physics. QCD is a quantum field theory that characterizes the interaction between particles such are the quarks and gluons. In high energy regime or for short distances, QCD shows the properties of asymptotic freedom, and in low energies, for long distances these particles are enclosed in groups of particles that are called hadrons.

In 1964 for the first time was admitted the quark model [1], which states that hadrons are composed of other particles that are called quarks. The quarks were discovered for the first time in experimental way, in 1970, in particle accelerators. Quarks are found permanently enclosed in hadrons or in neutral-colored groupings. The main application of QCD theory it is lattice QCD (LQCD) [2]. Also, exists other important phenomena and applications of Quantum Chromodynamics such are: the perturbative methods in gauge theories, the study of asymptotic freedom of quarks and more as are mentioned in [3].

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The formulation of Quantum Chromodynamics in a Euclidian lattice is the approach to solve the QCD theory in law energy regimes or in non-perturbative regimes. The lattice QCD is a lattice gauge filed theory that is adopted on a lattice with N points per directions. The dimensions are four, so the 3 dimensions of space and one of time. The lattice formulation is done in that way where in lattice sites are positioned the quarks fields and on the links between sites are positioned gluon fields.

The discrete format of the formulation of Quantum Chromodynamics theory in a lattice with Euclidian geometry insert a cut-off momentum of the scalinga^{-1}, with a that represent the lattice space. For consequence, when the lattice space goes to zero, we approach to the continuum Quantum Chromodynamics. So, finally, using nonperturbative methods, the lattice QCD can be defined exactly mathematically. In this way we have the possibility to measure physical quantities with high precision using numerical simulations. The lattice space it is very important to determine because it serves as an ultraviolet regulator and makes the quantum field theory not continuous.

The potential of two statics quarks, in distance R by each other, represents the energy that have the gauge fields along this distance. If the distance R is increasing the energy connecting two quarks begins to grow and it is impossible to use perturbative methods in order to calculate physical quantity. This phenomenon it is known as quark confinement, so quarks are impossible to be separated by each other, they exist only confine within other physical particle such are hadrons.

So, what is important to study it is the quark-antiquark potential for small distances when the lattice simulation of QCD theory gives accuracy results and the nonperturbative methods can be applied. In 1970 [4], the Creutz work to compute the potential between two static quarks showed and confirmed the fact that quarks cannot be detected alone but they are confined within hadrons, so as we can expect the quark-antiquark potential rise linearly at large distance. Other contributions in this direction are done in [5], [6], [7] and more recently in [14], [15], [16], [17], where it is confirmed the linear behavior of the potential for the intermediate distances. Based on these references in all representations of SU (3) it definitely true that quarks are confined within hadrons.

On the other hand, if we want to find physical quantity from lattice quantity, we need a physical quantity as reference, which we can easily compute from lattice QCD simulations and in the same time this quantity it is exactly determined from experimental QCD. The physical quantity that completes these conditions it is the string tension K along two static color quarks. So finally, we will calculate the lattice space a using as a trusted reference the physical quantity that we called string tension$K$.

The lattice parameter than can be used to convert nondimensional physical quantity that we take from lattice calculations into dimensional ones. The string tension K also can be derived in LQCD from Wilson loops. The computation on the lattice of K from Wilson loops have older references [8] [9], [10] [11]. Using these references as starting point we want to bring a new method that measure the quark-anti-quark potential between two static quarks using QCDLAB package, version 2.0 [12].

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2. MATERIALS AND METHODS

In lattice QCD calculations it very important to take dimensional results. For this purpose, we computed the lattice space a. In order to define the lattice space, we calculated the effective quark potential from Wilson loops with planar geometry and for three lattice volumes 84, 124, 164, using QCDLAB2 software. We propose here a new method to calculate the potential between two quarks. This potential can be extracted if we study the large time behaviour of the Wilson loops.

We build first a closed path C (R, T) which have rectangular geometry, where T is the time dimension. Then we construct the Wilson loops W (R, T), which mathematically are determined as the trace of path-ordered products of the link variables U_μ (n) that are directed along the line C (R, T).

The loop that it is constructed in this way gives the links connecting a quark-anti-quark pair at rest, that are in distance R from each other and that are displacing over the time T. Referring the Euclidian space-time geometry, for large T values, this observable will give the ground state energy. So, we start calculating the energy, and we have computed for different times, the correlation function of two statics quarks operators:

$$W(R,T) = \langle 0|O_R(0)O_R(T)^*|0\rangle = \langle O_R(0)O_R(T)^*\rangle \qquad (1)$$

$O_R(t)$ it is the operator which is gauges invariant and it is determined as:

$$O_R(t) = \bar{q}(t,0)U\big((t,0) \to (t,R)\big)q(t,R) \qquad (2)$$

When $U\big((t,0) \to (t,R)\big)$ is the link that represents the gauge field between two static quarks from site (t,0) to site (t, R). Thus, we have defined the loops of Wilson:

$$W(R,T) = \langle tr\, U\big((0,0) \to (0,R)\big)U\big((0,R) \to (T,R)\big)U\big((T,0) \to (T,R)\big)^* U\big((0,0) \to (T,0)\big)^*\rangle \qquad (3)$$

Also, we can write these loops as:

$$W(R,T) = \sum_{n\geq 1} c_n e^{-V_n(R)T} \qquad (4)$$

Where the expression $V_1(R) \equiv V(R)$ represents the state with minimal energy and the other states for n>1 represents the potentials in the excited states. As consequences, we have extracted potential supposing that:

$$W(R,T) \cong c_1 e^{-V(R)T}, \qquad (5)$$

And we have calculated the effective potentials from:

$$V(R)_{eff} = -\log\frac{W(R,T+1)}{W(R,T)} \qquad (6)$$

Further, we have done the fit of the effective potential, for different R, in accordance with the theoretical physical model

$$V(R)_{eff} = V_0 + KR + \frac{\alpha}{R} \qquad (7)$$

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In equation (7), the parameter α is a constant which represent the coefficient of the Coulomb term and the parameter K it is the string tension. Multiplying with lattice space the equation (7), we have taken a new form of the equation (7) but in lattice unit

$$\hat{V}(R)_{eff} = \hat{V}_0 + \hat{K}\frac{R}{\mathrm{a}} + \frac{\mathrm{a}}{R}\hat{\alpha}. \quad (8)$$

As we mentioned before we have done all simulations using QCDLAB2 program. QCDLAB version 2.0 it is the first software dedicated to lattice QCD simulations in four dimensions. It is built as a small package with a lot of short programs and algorithms that give possibilities to study the properties of QCD without using high processing computers and without develop dedicated algorithms. The author was Professor Artan Borici, who passed away from the Covid19 pandemic. What is more effective in QCDLAB2, comparing to other similar software, it is the fact that connect the linear operators of QCD to linear operators of the GNU Octave language. This property of QCDLAB gives the opportunity for develop and test efficient coding programs and also minimal times required for running these codes. We have calculated the string tension K repeating simulations for 100 different configurations, for 8^4, 12^4 and 16^4 lattice. The results taken are fitted with the equation (8). The R was chosen for fit it is between R = 0.5 and R = 8. The lattice physical volume (L^4) with length L per directions, it is equivalented to the lattice volume (N^4) with N point for direction from L= aN.

3. RESULTS AND DISCUSSION

We have done all simulations using Wilson action for 84, 124, 164 lattices with different background gauges fields, specifically for 100 gauges fields and testing for three different coupling constants, using QCDLAB2 soft.

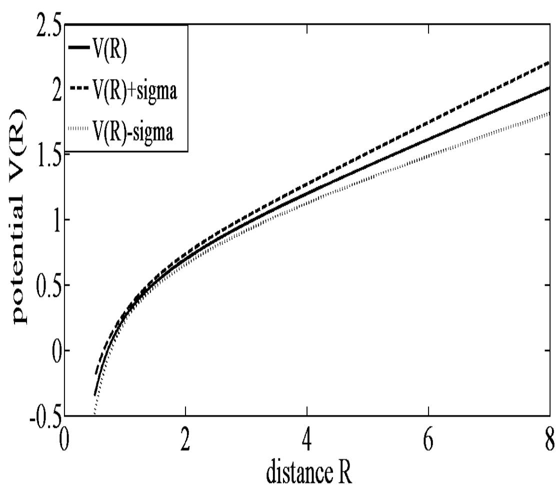

Fig 1: Graphical representation of potential between two static quarks in 8^4 lattice, in gauge field background with 5.7 coupling constant and dimensionless unit

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Fig 2: In Graphical representation of potential between two static quarks in 12^4 lattice, in gauge field background with 5.8 coupling constant and dimensionless unit

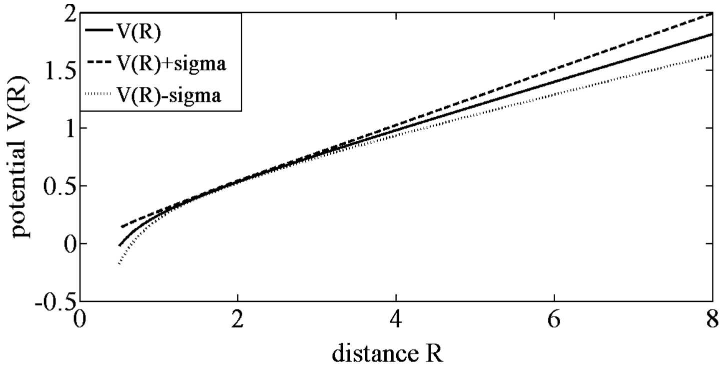
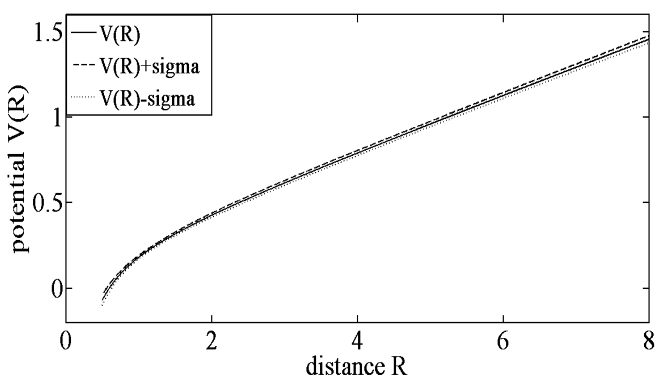


Fig 3: In Graphical representation of potential between two static quarks in 16^4 lattice, in gauge field background with 6 coupling constant and dimensionless unit



As we can see from Fig. 1, Fig. 2 and Fig. 3 of graphical representation of potential between two static quarks, for different coupling constant, we found that this potential behaves as a Coulomb–like term for small R distances from expression $V(R) \cong V_0 + \alpha/R$ and it rises linearly for large R distances from expression $V(R) \cong V_0 + KR$.

These graphical results that are taken using QCDLAB, confirms the known fact of confinements of quarks eternally within hadrons. The assessments of statistical errors for parameters of the equation (8) are made using Jackknife method [13]. This method that is exploiting to evaluate statistical errors it used in cases when we want to calculate statistical errors of derived quantity.

The numerical results together with their statistical errors of the parameters in equation (8), are given in Table 1.

Table 1: the calculated lattice space$a$, string tension $\hat{K}$, and their respective statistical errors 8^4, 12^4, 16^4 lattices

Lattice	The coupling constant parameter	The lattice space parameter (femtometer)	The string tension parameter (lattice unit)	The error of lattice space	The error of string tension
8^4	5.7	0.2011(15)	0.1670(79)	1.1003e-04	2.54-03
12^4	5.8	0.1630(07)	0.1592(15)	9.0097e-05	2.67e-03
16^4	6	0.1095(73)	0.1410(27)	1.0980e-05	2.84e-04

Referring numerical results, tabulated in Table 1, we have the values of the lattice space parameter and the values of the string tension parameter for 8^4, 12^4, 16^4 lattices. Also, in this table are represented the statistical errors of the lattice space parameter and the string tension parameter using Jackknife method. As we can see from these numerical results the values of lattice space and string tension are taken within the statistical error rate of calculated. Specifically, for lattice 8^4 the lattice space parameter is a = [0.2011(15) ± 1.1003×10^{-4}] (fm) and the string tension is K = [0.1670(79) ± 2.67×10^{-3}] (in lattice unit). The same analyze can be done for other lattice volumes.

4. CONCLUSIONS

The lattice space for different lattice volumes and for different coupling constant can be defined from Wilson loops in lattice QCD simulations. For lattice volume 8^4, the lattice scale is a = 0.2023, for lattice volume 12^4 the lattice scale is a = 0.1846 and for lattice volume 16^4 the lattice scale is a = 0.2023. In this way, using simple and very effective software we found the lattice scale for different lattice volume. If we have lattice space, we can find all physical quantities from dimensionless quantity in physical quantity, with their respective physical unit. From the graphical results, represented in Figure 1, Figure 2 and Figure 3 we found that the potential between two static quarks shows an important feature of QCD in low energy regimes, such as confinement of quarks. This expected result promotes the effectiveness of QCDLAB2. One of these purposes for our group will be the development of simulation and inversion algorithms that gain time of calculations. The QCDLAB version 2.0, in difference from the first version 1.0, is implemented for simulations in four dimensions, so for lattice QCD simulations in SU (3) gauge field. In our future work, to take more accurate results we have to collect and include more Wilson loops in our simulations. Also, if we want the more accuracy in determining the lattice space, we have to make simulations at larger values of R. So, this will require larger lattice volumes and on the other hand huge statistics works to do. Finally, we can conclude that, the QCDLAB2 it is a very promising software for this purpose.

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