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# EXTENDABLE BANHATTI SOMBOR INDICES FOR MODELING CERTAIN COMPUTER NETWORKS

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**ABSTRACT:**
Most networks involved in different fields are not of a single type, but rather a combination of two or more networks. These types of networks are called bridge networks which are used in interconnection networks of PC, mobile networks, the spine of the internet, networks involved in robotics, power generation interconnection, bio-informatics and chemical compound structures. Any number that can be uniquely determined by a graph is called graph invariants. During the recent two decades' innumerable numerical graph invariants have been described and used for correlation analysis. Till now no dependable assessment has been embraced to choose, how much these invariants are connected with a network graph or molecular graph. In this paper, it will talk about three distinct variations of bridge networks with great capability of expectation in the field of computer science, chemistry, physics, drug industry, informatics and mathematics in setting with physical and synthetic constructions and networks, since Banhatti Sombor (BSO) invariants are newly introduced and have various forecast characteristics for various variations of bridge graphs or networks. The review settled the topology of bridge graph/networks of three unique sorts with two types of BSO indices. These concluded outcomes can be utilized for the modeling of the above-mentioned networks.

**KEYWORDS**: Bridge networks; invariants; Banhatti Sombor indices; maple; network graph; molecular graph

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## 1. INTRODUCTION

A network of single topology performed well in the field of computer science and others like chemistry, bioinformatics and physics. But in the case of a merger of two or more networks in the form of a bridge network the efficiency and effectiveness are compromised [1]. The purpose of this study is to evaluate the topology of bridge networks before developing and using them in any field. Bridge graphs [2]are introduced by T. Mansour and M. Schork, which are a mix of networks bridged together in a single network. A bridge graph is a graph acquired from several graphs G1, G2, G3,...Gm by partner the vertices vi and vi + 1 by an edge∀, I = 1,2,..., m − 1. Then again, V.R. Kulli in 2016, define the possibility of various types of BSO invariants. Another vertex degree-based invariant graph named BSO index is utilized to catch the sharp lower and upper bounds of the associated networks and the attributes of the network arriving at the boundaries. There are two variations of BSO indices, the first one is the First Banhatti Sombor (BSO1) invariant and the second one is the Second Banhatti Sombor (BSO2) invariant [3]. A BSO index is a topological invariant which is a number associated with a network graph [4] that catches the symmetry of the network structure and gives a logical language to anticipating the attributes of the network.

As one more emerging science is created with the assistance of computer science, math and chemistry called cheminformatics, whose huge fragments consolidate Quantitative structure-activity relationship (QSAR) and Quantitative structure-property relationships (QSPR) and the portions can include the assessment physicochemical qualities of engineered blends [5].

Its first eminent application in chemistry was the examination of paraffin edges bubbling over by wiener. Different topological indices were introduced following this assessment that explained physicochemical properties. The progress of enormous scope incorporated circuit advancement has engaged the improvement of complex interconnected networks. Graph theory gives a vital mechanical assembly to planning and assessing such networks. Interconnected networks and graph theory give a point-by-point perception of these associated networks. These topological indices or invariants are additionally numeric qualities related to computer networks, their interconnections and their properties and so forth this assessment gives a premise to understand the significant topologies of some significant bridge networks and how these networks can be created based on the best topological properties. This highlight likewise gives possible help to researchers to consider network qualities better. Additionally, assuming that the connected networks are supplanted by various networks, this study can likewise compute and get the contrasting equations.

Computer networks from intranet to overall networks, electric power interconnection, social networks, robotics interconnected networks, the sexual ailment of networks of transmission, and genome networks are comparable with graph theory with the assistance of complex networks analysis apparatus. These multitudes of networks are at the top level of their utilization and differentiated. In this heap of cases, this study can figure boundaries called Topological invariants (TIs) that numerically portrayed the connectedness plans (structure) between the centers or performers in a network. So this study can build a mind-stunning network of general arrangements of regulations accomplice regulations (centers) that direct ordinary natural subjects for example. QSAR and QSPR are giving the establishment to these models. The last comment is that the use of the estimation in the network plane works with a quantitative assessment of different geography shielding planning calculations [6].

This paper initially presents the issue proclamation with bridge graph and BSO indices, also audit the concerned literature, thirdly talks about targets, importance, research gap and technique in research approach area, in the fourth segment dissect information and in the last segment compose results and finish up the examination. The review has suggestions in the fields of the computer industry, electronics, chemical industry, math, robotics and bioinformatics for modeling reasons for networks, network interconnections, power generation interconnection networks and chemical compounds. BSO topological invariants permit us to gather data about arithmetical structures and numerically foresee stowed away properties of different structures, for example, bridge networks.

## 2. LITERATURE REVIEW

The Materials Topological invariants are widely utilized for laying out the connection between the nanostructure and their physical-substance properties. The development of new nanostructures gives a feature to industry, gadgets, drugs, clinical medicines, correspondence, data and food science, etc. In this paper, sombor index is tried with physical-compound properties of octane isomers like entropy, acentric component, enthalpy of vaporization (HVAP) and standard enthalpy of vaporization (DHVAP) utilizing the straight models. The sombor index shows a brilliant relationship with these compound properties and extraordinarily high relationship with DHVAP. [7]. Chemical graph theory gives a connection between molecular properties and a molecular graph. The study likewise presents the M-polynomial of the silicate network and afterward, formulas of topological indices are applied to the silicate network[8].

Chemical Graph Theory is a part of Mathematical Chemistry that importantly affects the development of the Chemical Sciences. We present the contraharmonic-quadratic index (CQI) of a molecular graph. In this paper, we determine the CQI of some standard classes of graphs. We likewise compute the CQI of certain important nanostar dendrimers [9]. The study figures out the adjusted first and second BSO indices and symphonious BSO index of a few standard graphs, TUC4C8[p, q] nanotubes and TUC4[p, q] nanotubes[10] [11].

From the perspective of study, the progress of enormous scope integrated circuit advancement has to understand the improvement of complex interconnected networks. Graph theory gives a critical contraption to planning and assessing such networks. Interconnected networks and graph theory give an itemized cognizance of these

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associated networks. It computes the different topological indices of unpredictability based on the paired tree up to the k-level. The concluded results of a paper can be utilized for computer networks and chemical networks in topological portrayal [12].

The estimation of the irregularity indices of honeycomb networks, hexagonal networks, oxide networks, and silicate networks is finished. The outcomes are extremely useful in understanding the conduct of various computer networks and chemical networks. After understanding these expressions various scientists can develop their own best networks in the chemical and computer industries likewise [13]. Further, the study explains that graph theory is a field through which they compute topological indices for tracking down the properties of various chemicals and networks without playing out any sorts of investigations on them. It very well might be understanding just numerical expressions or conditions which are reasoned for them. It likewise works out topological indices for m-polynomial square shift networks which is a pieces of various chemical compounds with the assistance of division of edge [14].

It is resolved that it can tackle this present reality issue just by displaying the conduct of the issue by applying Chemical response network theory (CRNT). There are parcels of uses of chemical graph theory in bioscience and hypothetical science. It is inferred networks utilized in various chemical compounds and computer science and work out their topological indices for the first type, second type, and third type. They said that these outcomes are a lot of viable in the arrangement of new medications and accommodating in understanding the properties of chemical compounds [15]. Simonraj et al. portrayed the chemical network and is shown mathematically by topological indices in chemical graph theory. Then again, explicit physicochemical properties are related to essential chemical compounds. The graph accepts a basic part in showing and arranging any chemical network. The study decided on one more kind of graph, which is named the third type of hex-construed networks [16].

It is characterized that chemical response network theory is a space of applied number juggling that endeavors to show the direction of veritable chemical structures. Since its establishment during the 1960s, it has attracted a making research neighborhood, for the most part, because of its applications in normal science and speculative science [17]. It has likewise attracted income from unadulterated mathematicians considering the enrapturing issues that ascent up out of the mathematical plans included. In this report, the process portrayed topological indices; explicitly, calculating the numerical index, SK index, 1 SK index, and 2 SK index of the octagonal network. They in like way figure out the whole availability index and changed Randić index of the subordinate network [18]. These indices of honeycomb networks are especially powerful to understand the physicochemical properties of chemicals. These real factors may be significant for people working in computer programming and science who experience honeycomb networks. An ideal level of a particular index can be procured by placing a constraint on n [19].

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Another review depicted that the m-polynomial is one of the arithmetical polynomials that are useful in theoretical chemistry. It plays significant work in handling the particular explanations of various degree-based topological indices. They got various degree-based topological indices for benzene rings introduced in the p-type-surface network and Tickysim spinnaker model (TSM) sheet [20]. Further, computations of topological indices of confined pent-heptagonal nano-sheet are finished. QSARs address perceptive models got from the utilization of verifiable mechanical assemblies connecting with the natural activity (counting charming helpful effects and appalling side effects) of engineered materials (drugs/harms/biological poisons) with descriptors illustrative of molecular structure as well as properties [21].

It is delineated that Hand gestures recognition (HGR) is one of the essential spaces of examination for Human-computer interaction (HCI) applications. Most existing philosophies rely upon neighborhood or numerical properties of pixels. Regardless, there are a few certifiable hardships on HGR systems like affectability to upset, scale, lighting up, trouble, and hindrance. There are utilized a particular and wide-range dataset with 31 kinds of tokens of various amounts of fingers with raising disengagement. These are absolute 2170 pictures. Since the GNG outline isn't exceptional, they also attempted the affectability of our calculation to this graph [22].

As indicated by readings clarification that another emerging science is cheminformatics which is a blend of chemistry, mathematics and computer science. It is the principal concern and constituent part is QSAR and QSPR which essentially investigate physicochemical properties of chemicals and their structures. This paper kept an eye on the OTIS exchanged networks and bi-traded networks and examined their topological indices. They determined complete randi ć, general total accessibility, first and second zagreb, first and second hyper zagreb, hyper zagreb atom bond and numerical indices for both the gathering of networks by considering the reason network as way Pn and k-standard graph Rk. They similarly gave express formulae for ABC4 and GA5 indices of these networks with the reason Rk network [23].

In all of these cases, they can register boundaries considered TIs that numerically portrayed the connectedness plans between the hubs or performers in a network. Accordingly, TIs are useful as commitments for QSPR models at every underlying level. Without a doubt, even broad arrangements of regulations may be advanced toward using figuring and information systems like networks. So they can foster a flighty network of generally speaking arrangements of regulations partner regulations (hubs) that oversee fundamental natural focuses for instance. On the other hand, a precise legal structure is relied upon to provide legitimate and relevant guidance to addressing different enlisting methodologies as applied to coherent examination [24].

In this paper, researchers focused on different kinds of issues related to graph theory and their executions and ideas in the field of programming to show the ampleness of graph theory. These applications are familiar particularly with loosening up graph theory and showing its goal and importance in network designing. This paper is wanted to help the under examinations of programming to get important information on graph theory and its importance with various subjects like working systems, networks, data sets, software engineering, and so on this paper zeroed in on the different employments of colossal graph theory that have congruity to the field of computer programming and applications [25]. The speculative contemplations of the graph are particularly used by computer programming applications. Particularly in research spaces of programming such as data mining, picture division, gathering, picture getting, networking, and so forth For instance, a data configuration can be sorted out as a tree which thus used vertices and edges. Moreover, modeling of network geographies should be possible by utilizing graph considerations [26].

Consequently, these fields have braced the improvement of different new outline speculative considerations and prompted many testing graph theory issues. They can expect that the continued trade between graph theory and different spaces of utilization will incite basic new turns of events. The basic control of graph theory in PC applications is the improvement of graph assessments [27]. Topological invariants engage us to assemble data about logarithmic structures and give us a numerical strategy to figure out the secret properties of various structures. Various strategies are available in history to check the nature of a topological index. There are two principal conflicts of topological indices, first one is the degree-based topological and the below-average is known as distance-based topological indices. There are many such invariants are available in history [28]. BSO index and its other variant have great capability of expectation in the field of computer science, math, chemistry, drugs, informatics and power generation in setting with physical and chemical structures and networks[29].

## 3. RESEARCH METHODOLOGY

### 3.1. Objectives

The principal objective of this study is to research the topological invariants of bridge computer networks. The review figures out the force of reality of topological indices in bridge networks like workstation networks, interconnection networks of processors, power interconnection networks and chemical structures and so forth. This paper study clarifies the BSO indices, and their different forms and their advantages. This article clarifies and is mindful of the ongoing utilization of BSO indices. Its great goal is to foster formulas, so it can check the topology, and execution of bridge networks without doing/performing tests, likewise before assembling them and modeling bridge networks [30] [31].

### 3.2. Hypothesis

The reading considers the accompanying hypothesis for the advancement of bridge networks utilized in computer networks, interconnection networks of processors, power interconnection networks, chemical structures and mechanical technology

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interconnection networks since manufacturers and engineers need to understand the complexity and force of execution and disappointment-free products.

### 3.3. Method

This deliberate study will take a current bridge network, partner it with a graph and address the topology of the graph with the assistance of BSO indices and its other forms. The concluded results about the type of numeric expression will contrast with existing outcomes. These concluded outcomes will be pertinent to numerous different networks in the fields of computer networks, processor interconnection networks, memory interconnection networks, power interconnection networks, and picture handling subsequently. This model is especially unsettling as it tackled the topology of a bridged network in numeric and graphical structure and gives exact outcomes. After analysis, a recreation apparatus maple is utilized for the confirmation and approval of results.

## 4. EXPERIMENTAL RESULTS

A bridge graph is a graph obtained from many graphs G1, G2, G3,...Gm by associate the vertices vi and vi + 1 by an edge∀, i = 1,2,..., m − 1 [32]. BSO indices have two main forms $BSO_1$ indices and the BSO2 indices with different variants [33].

$$BSO_1(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{d_u^2} + \frac{1}{d_v^2}} \quad (1)$$

$$RBSO_1(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{(d_u-1)^2} + \frac{1}{(d_v-1)^2}} \quad (2)$$

Eq. (1) and Eq. (2) show the BSO1 index and its reduced form which will be used for the solution of bridge networks mentioned in Fig. 2, Fig. 5, and Fig. 8.

$$BSO_2(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\frac{1}{d_u^2} + \frac{1}{d_v^2}}} \quad (3)$$

$$RBSO_2(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\frac{1}{(d_u-1)^2} + \frac{1}{(d_v-1)^2}}} \quad (4)$$

Eq. (3) and Eq. (4) show the BSO2 index and its reduced form which will be used for the solution of bridge networks mention in Fig. 3, Fig. 6, and Fig. 9.

Edge partitions of Gr (Ps, v) over Ps are ε(1 ,1), ε(2 ,2), ε(2 ,3) and ε(3 ,4) and their frequencies are R, 3r + 2, R and r - 3 respectively. This is a description of the edge partitions of graph Gr (Ps, v) over Ps of the bridge graph given in Fig. 2.

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### 4.1 Main Results

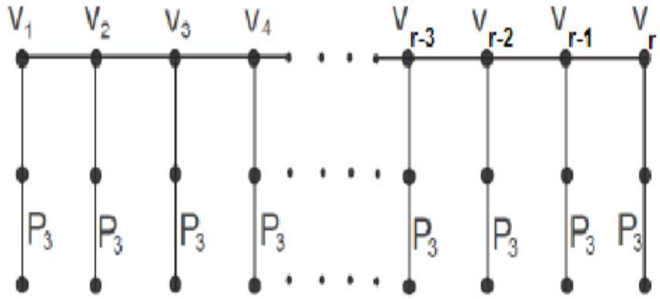


**Figure 1: Gr (Ps, v) over Ps for Bridge Network**

Fig. 1 shows bridge networks in which bus networks and star networks bridge in a tree-like structure.

#### 4.1.1 Bridge Graph Gr (Ps, v) Over Path

If the vertex set is V then by the observation of Figure 2, it can order this vertex set into four subsetsV1, V2, V3 and V4, Such that V=V1+V2+V3+V4. If E represents the edge set. Fig. 2 shows that there are four distinct kinds of edges present in the graph bridge graph Gr (Ps, v) over the path of hybrid networks.

#### 4.1.2 Theorem 1

Let G be a graph of Gr (Ps, v) over Ps, then, BSO1, R BSO1, BSO2, and R BSO2 indices are:

$$BSO_1(G) = \sqrt{2}\,(r) + \frac{1}{2}\sqrt{2}(3\,r + 2) + \frac{1}{6}\sqrt{13}\,r + \frac{5}{12}r - \frac{5}{4} \quad (5)$$

$$RBSO_1(G) = \sqrt{2}(3\,r + 2) + \frac{1}{2}\sqrt{5}\,r + \frac{1}{6}\sqrt{13}\,(r - 3) \quad (6)$$

$$BSO_2(G) = \frac{1}{2}\sqrt{2}\,(r) + \sqrt{2}(3\,r + 2) + \frac{6}{13}\sqrt{13}\,r + \frac{12}{5}r - \frac{36}{5} \quad (7)$$

$$RBSO_2\,(G) = \frac{1}{2}\sqrt{2}\,(3r + 2) + \frac{2}{5}\sqrt{5}r + \frac{6}{13}\sqrt{13}(r - 3) \quad (8)$$

Eq. (5), Eq. (6), Eq. (7) and Eq. (8) represent the proven results of the graph of Gr (Ps, v) over Ps mentioned in Fig. 2.

#### 4.1.3 Investigation of Bridge Graphs by BSO1 Indices and its reduced form

Proof

$$BSO_1(G) = \sum_{uv \in E(G)} \sqrt{\frac{1}{d_u^2} + \frac{1}{d_v^2}}$$

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E-Publication Online Open Access
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$$BSO_1(G)= \sqrt{\frac{1}{1^2}+\frac{1}{1^2}}\, r + \sqrt{\frac{1}{2^2}+\frac{1}{2^2}}\,(3\,r+2) + \sqrt{\frac{1}{2^2}+\frac{1}{3^2}}\, r + \sqrt{\frac{1}{3^2}+\frac{1}{4^2}}\,(r-3)$$

$$BSO_1(G) = \sqrt{2}\,(r) + \frac{1}{2}\sqrt{2}(3\,r+2) + \frac{1}{6}\sqrt{13}\, r + \frac{5}{12} r - \frac{5}{4}$$

$$RBSO_1(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{(d_u-1)^2}+\frac{1}{(d_v-1)^2}}$$

$$RBSO_1(G) = \sqrt{\frac{1}{(2-1)^2}+\frac{1}{(2-1)^2}}\,(3\,r+2) + \sqrt{\frac{1}{(2-1)^2}+\frac{1}{(3-1)^2}}\, r + \sqrt{\frac{1}{(3-1)^2}+\frac{1}{(4-1)^2}}\,(r-3)$$

$$RBSO_1(G) = \sqrt{2}(3\,r+2) + \frac{1}{2}\sqrt{5}\, r + \frac{1}{6}\sqrt{13}\,(r-3)$$

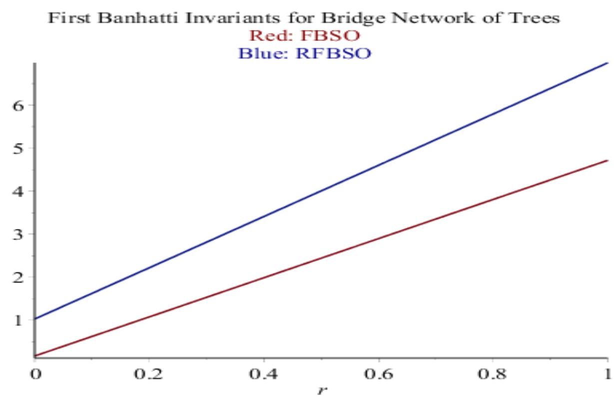


**Figure 2: $BSO_1$ and $RBSO_1$ for Gr (Ps, v) over Ps**

Fig. 2 shows the results Eq. (1) & (2) of BSO1 indices and their reduced form in red and blue lines respectively.

### 4.1.4 Investigation of Bridge Graphs by $BSO_2$ Indices and its reduced form

Proof

$$BSO_2(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\frac{1}{d_u^2}+\frac{1}{d_v^2}}}$$

$$BSO_2(G) = \frac{1}{\sqrt{\frac{1}{1^2}+\frac{1}{1^2}}}\, r + \frac{1}{\sqrt{\frac{1}{2^2}+\frac{1}{2^2}}}\,(3\,r+2) + \frac{1}{\sqrt{\frac{1}{2^2}+\frac{1}{3^2}}}\, r + \frac{1}{\sqrt{\frac{1}{3^2}+\frac{1}{4^2}}}\,(r-3)$$

$$BSO_2(G) = \frac{1}{2}\sqrt{2}\,(r) + \sqrt{2}(3\,r+2) + \frac{6}{13}\sqrt{13}\, r + \frac{12}{5} r - \frac{36}{5}$$

$$RBSO_2(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\frac{1}{(d_u-1)^2}+\frac{1}{(d_v-1)^2}}}$$

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$$RBSO_2\ (G) =$$

$$\frac{1}{\sqrt{\frac{1}{(1-1)^2}+\frac{1}{(1-1)^2}}}\,r + \frac{1}{\sqrt{\frac{1}{(2-1)^2}+\frac{1}{(2-1)^2}}}\,(3\,r+2) + \frac{1}{\sqrt{\frac{1}{(2-1)^2}+\frac{1}{(3-1)^2}}}\,r + \frac{1}{\sqrt{\frac{1}{(3-1)^2}+\frac{1}{(4-1)^2}}}\,(r-3)$$

$$RBSO_2\ (G) = \ \frac{1}{2}\sqrt{2}\,(3r+2) + \frac{2}{5}\sqrt{5}r + \frac{6}{13}\sqrt{13}(r-3)$$

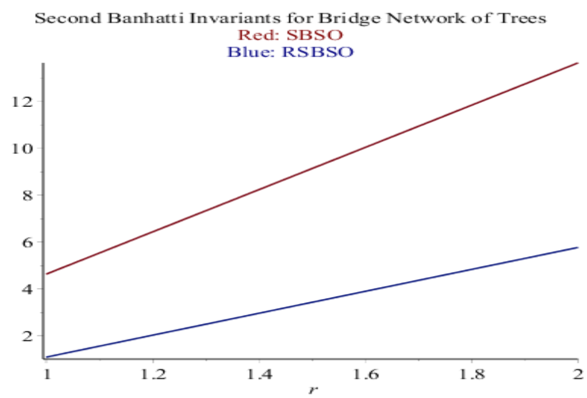


**Figure 3: $BSO_2$ and $RBSO_2$ for Gr (Ps, v) over Ps**

The edge partitions of graph Gr (Ks, v) Over Ks of the bridge graph given in Fig. 5 are ε(2 ,2), ε(2 ,3), ε(2 ,4), ε(3 ,5) and ε(4 ,6) with frequencies rs – 2r, 2r – 4, 2 and r-3 respectively.

### 4.2 Main Results

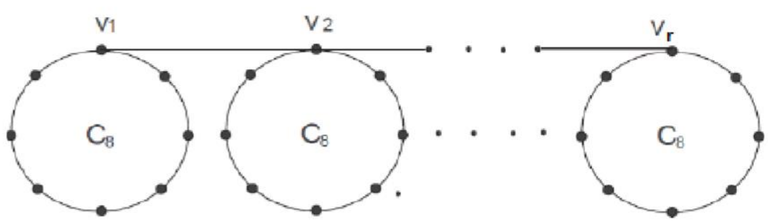


**Figure 4: Gr (Cs, v) over Cs for Bridge Network**

Fig. 4 shows the bridge networks in which bus networks and ring networks bridge together [34].

#### 4.2.1 Bridge Graph Gr (Cs, v) Over Cycle

Assuming V is the arrangement of vertices seen in Figure 4, this arrangement of vertices can be parted into four subsets V1, V2, V3, and V4 to V=V1+V2+V3+V4. When ε (du , dv) addresses an edge set. Fig. 4 shows a half-and-half network cycle with five distinct kinds of edges in the network graph of the bridge graph Gr (Cs, v) [35].

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### 4.2.2 Theorem 2

Let G be a graph of Gr (Cs, v) over Cs Then BSO1, RBSO1, BSO2, and R BSO1 indices are:

$$BSO_1(G) = \frac{1}{2}\sqrt{2}\,(rs-2r) + \frac{2}{3}\sqrt{13} + \frac{1}{4}\sqrt{5}(2\,r-4) + \frac{2}{15}\sqrt{34} + \frac{1}{12}(r-3)\sqrt{13} \quad (9)$$

$$RBSO_1(G) = \sqrt{2}\,(rs-2r) + \frac{5}{2}\sqrt{5} + \frac{1}{3}\sqrt{10}(2r-4) + \frac{1}{15}\sqrt{34(}r-3) \quad (10)$$

$$BSO_2\,(G) = \sqrt{2}\,(rs-2r) + \frac{24}{13}\sqrt{13} + \frac{4}{5}\sqrt{5}(2r-4) + \frac{15}{17}\sqrt{34(}r-3) + \frac{12}{13}\sqrt{13(}r-3) \quad (11)$$

$$RBSO_2\,(G) = \frac{1}{2}\sqrt{2}\,(3r+2) + \frac{2}{5}\sqrt{5}r + \frac{3}{10}\sqrt{10}(r-3) + \frac{4}{5}\sqrt{5}(r-3) + \frac{15}{34}\sqrt{34}(r-3) \quad (12)$$

Eq. (9), Eq. (10), Eq. (11) and Eq. (12) represent the proven results of the graph of Gr (Cs, v) over Cycle mentioned in Fig. 4.

### 4.2.3 Investigation of Bridge Graphs by BSO1 Indices and its reduced form

Proof

$$BSO_1(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{d_u^2} + \frac{1}{d_v^2}}$$

$$BSO_1(G) = \sqrt{\frac{1}{2^2}+\frac{1}{2^2}}(rs-2r) + \sqrt{\frac{1}{2^2}+\frac{1}{3^2}}(4) + \sqrt{\frac{1}{2^2}+\frac{1}{4^2}}(2r-4) + \sqrt{\frac{1}{3^2}+\frac{1}{5^2}}(2) + \sqrt{\frac{1}{4^2}+\frac{1}{6^2}}(r-3)$$

$$BSO_1(G) = \frac{1}{2}\sqrt{2}\,(rs-2r) + \frac{2}{3}\sqrt{13} + \frac{1}{4}\sqrt{5}(2\,r-4) + \frac{2}{15}\sqrt{34} + \frac{1}{12}(r-3)\sqrt{13}$$

$$RBSO_1(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{(d_u-1)^2} + \frac{1}{(d_v-1)^2}}$$

$$RBSO_1(G) =$$

$$\sqrt{\frac{1}{(2-1)^2}+\frac{1}{(2-1)^2}}(rs-2r) + \sqrt{\frac{1}{(2-1)^2}+\frac{1}{(3-1)^2}}(4) + \sqrt{\frac{1}{(2-1)^2}+\frac{1}{(4-1)^2}}(2r-4) + \sqrt{\frac{1}{(3-1)^2}+\frac{1}{(5-1)^2}}(2) + \sqrt{\frac{1}{(4-1)^2}+\frac{1}{(6-1)^2}}(r-3)$$

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
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Vol: 41 Issue: 11-2022
DOI 10.17605/OSF.IO/U9P3B

$$RBSO_1(G) = \sqrt{2}\,(rs - 2r) + \frac{5}{2}\sqrt{5} + \frac{1}{3}\sqrt{10}(2r - 4) + \frac{1}{15}\sqrt{34}(r - 3).$$

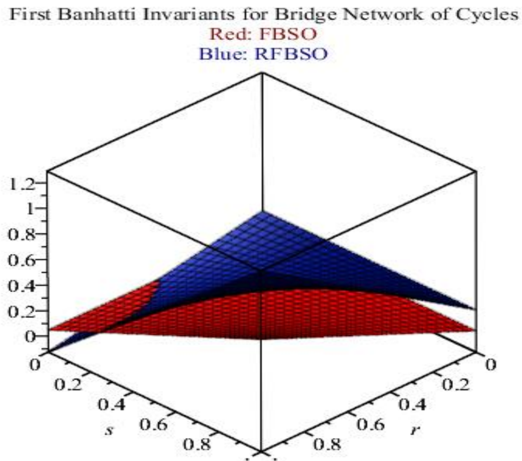


**Figure 5: $BSO_1$ and $RBSO_1$ for Gr (Cs, v) over Cycle**

Fig. 5 shows the results Eq. (1) & (2) of BSO1 indices and its reduced form in red and blue colors respectively in the 3D version.

### 4.2.4 Investigation of Bridge Graphs by BSO2 Indices and its reduced form

Proof

$$BSO_2(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\frac{1}{d_u^2}+\frac{1}{d_v^2}}}$$

$$BSO_2(G) = \frac{1}{\sqrt{\frac{1}{2^2}+\frac{1}{2^2}}}(rs - 2r) + \frac{1}{\sqrt{\frac{1}{2^2}+\frac{1}{3^2}}}(4) + \frac{1}{\sqrt{\frac{1}{2^2}+\frac{1}{4^2}}}(2r - 4) + \frac{1}{\sqrt{\frac{1}{3^2}+\frac{1}{5^2}}}(2) + \frac{1}{\sqrt{\frac{1}{4^2}+\frac{1}{6^2}}}(r - 3)$$

$$BSO_2(G) = \sqrt{2}\,(rs - 2r) + \frac{24}{13}\sqrt{13} + \frac{4}{5}\sqrt{5}(2r - 4) + \frac{15}{17}\sqrt{34}(r - 3) + \frac{12}{13}\sqrt{13}(r - 3)$$

$$RBSO_2\,(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\frac{1}{(d_u-1)^2}+\frac{1}{(d_v-1)^2}}}$$

$$RBSO_2\,(G) = \frac{1}{\sqrt{\frac{1}{(2-1)^2}+\frac{1}{(2-1)^2}}}(rs - 2r) + \frac{1}{\sqrt{\frac{1}{(2-1)^2}+\frac{1}{(3-1)^2}}}(4) + \frac{1}{\sqrt{\frac{1}{(2-1)^2}+\frac{1}{(4-1)^2}}}(2r - 4) + \frac{1}{\sqrt{\frac{1}{(3-1)^2}+\frac{1}{(5-1)^2}}}(2) + \frac{1}{\sqrt{\frac{1}{(4-1)^2}+\frac{1}{(6-1)^2}}}(r - 3)$$

$$RBSO_2\,(G) = \; = \frac{1}{2}\sqrt{2}\,(3r + 2) + \frac{2}{5}\sqrt{5}r + \frac{3}{10}\sqrt{10}(r - 3) + \frac{4}{5}\sqrt{5}(r - 3) + \frac{15}{34}\sqrt{34}(r - 3).$$

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
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Vol: 41 Issue: 11-2022
DOI 10.17605/OSF.IO/U9P3B

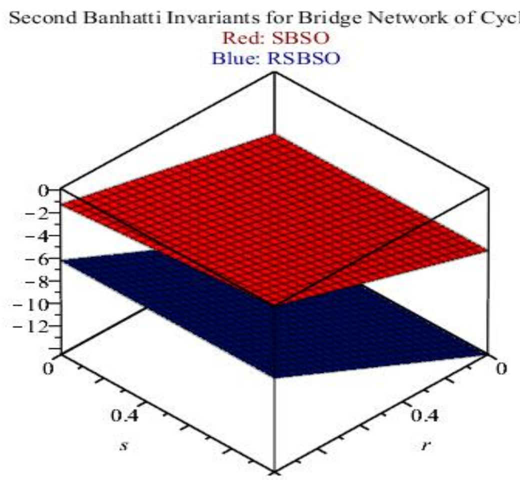


**Figure 6: $BSO_2$ and $RBSO_2$ for Gr (Cs, v) over Cs**

Fig. 6 shows the results Eq. (3) & (4) of BSO2 indices and their reduced form in red and blue colors respectively in the 3D version.

The edge partitions of graph Gr (Ks, v) Over Ks of the bridge graph given in Fig. 8 with the number of occurrences.

### 4.3 Main Results

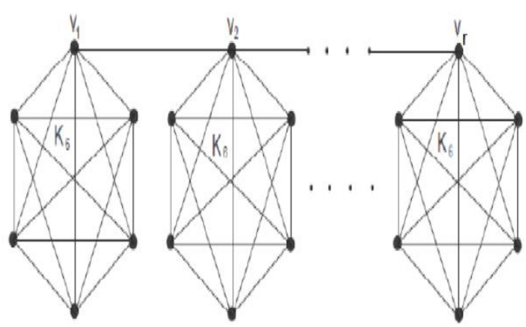


**Figure 7: Gr (Ks, v) over Ks**

Fig. 7 shows the bridge networks in which bus networks and fully connected networks are bridged together.

#### 4.3.1 Bridge Graph Gr (Ks, v) Over Complete Graph

Assuming that the vertices set are V, understanding Fig. 6 allows us to sort this set of vertices into three subsets V1, V2, and V3 so that V=V1+V2+V3. If E shows the edge set, figure 6 shows the bridge graph Gr (Ks, v) of the complete graph of the hybrid network [36]. The bridge graph of the network graph has five different edges. Tab. 3 provides a detailed description of the edge set [37].

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
ISSN：1671-5497
E-Publication Online Open Access
Vol: 41 Issue: 11-2022
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### 4.3.2 Theorem 3

Let G be a graph of Gr (Ks, v) over Ks. Then BSO1, RBSO1, BSO2, and RBSO2 indices are

$$BSO_1(G) = \frac{1}{10}\sqrt{41} + \sqrt{1+\frac{16}{(s-1)^2}} + \frac{1}{5}\sqrt{2}(r-2) + \frac{1}{5}\sqrt{1+\frac{16}{(s-1)^2}}(r-2) + \sqrt{2}\sqrt{\frac{1}{(s-1)^2}} + (\frac{1}{2}rs(r-1) - r - 1) \quad (13)$$

$$RBSO_1(G) = \frac{5}{6} + \frac{2}{3}\sqrt{1+\frac{9}{(s-2)^2}} + \frac{1}{4}\sqrt{2}(r-2) + \frac{1}{2}\sqrt{1+\frac{16}{(s-1)^2}}(r-2) + \sqrt{2}\sqrt{\frac{1}{(s-2)^2}}(\frac{1}{2}rs(r-1) - r - 1) \quad (14)$$

$$BSO_2(G) = \frac{40}{41}\sqrt{41} + \frac{8}{\sqrt{1+\frac{16}{(s-1)^2}}} + \frac{5}{2}\sqrt{2}(r-2) + \frac{5}{\sqrt{1+\frac{25}{(s-1)^2}}}(r-2) + \frac{1}{4\sqrt{\frac{1}{(s-1)^2}}}\sqrt{2}\,(\frac{1}{2}rs(r-1) - 2r - 2) \quad (15)$$

$$RBSO_2(G) = \frac{24}{5} + \frac{6}{\sqrt{1+\frac{9}{(s-2)^2}}} + 2\sqrt{2}(r-2) + \frac{4}{\sqrt{1+\frac{16}{(s-2)^2}}}(r-2) + \frac{1}{4\sqrt{\frac{1}{(s-1)^2}}}\sqrt{2}\,(\frac{1}{2}rs(r-1) - 2r - 2) \quad (16)$$

Eq. (13), Eq. (14), Eq. (15) and Eq. (16) represent the proven results of the graph of Gr (Ks, v) over the complete graph mentioned in Fig. 7.

The graph of Gr (Ks, v) over the complete graph has edge partitions ε(4 ,7), ε(4 ,s+1), ε(5 ,8), ε(5 ,s+2) and ε(5 ,s+2) with frequencies 2, 2, r-2, r-2 and [rs(r-1)-2(r+1)] /2 respectively.

### 4.3.3 Investigation of Bridge Graphs by $BSO_1$ Indices and its reduced form

Proof

$$BSO_1(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{d_u^2} + \frac{1}{d_v^2}}$$

$$BSO_1(G) = \sqrt{\frac{1}{4^2}+\frac{1}{7^2}}(2) + \sqrt{\frac{1}{4^2}+\frac{1}{(s+1)^2}}(2) + \sqrt{\frac{1}{5^2}+\frac{1}{8^2}}(r-2) + \sqrt{\frac{1}{5^2}+\frac{1}{(s+2)^2}}(r-2) + \sqrt{\frac{1}{(s-1)^2}+\frac{1}{(2s-4)^2}}(rs(r-1) - 2(r+1))/2$$

$$BSO_1(G) = \frac{1}{10}\sqrt{41} + \sqrt{1+\frac{16}{(s-1)^2}} + \frac{1}{5}\sqrt{2}(r-2) + \frac{1}{5}\sqrt{1+\frac{16}{(s-1)^2}}(r-2) + \sqrt{2}\sqrt{\frac{1}{(s-1)^2}} + (\frac{1}{2}rs(r-1) - r - 1)$$

$$RBSO_1(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{(d_u-1)^2} + \frac{1}{(d_v-1)^2}}$$

$$RBSO_1(G) = \sqrt{\frac{1}{(4-1)^2}+\frac{1}{(7-1)^2}}(2) + \sqrt{\frac{1}{(4-1)^2}+\frac{1}{((s+1)-1)^2}}(2) + \sqrt{\frac{1}{(5-1)^2}+\frac{1}{(8-1)^2}}(r-2) + \sqrt{\frac{1}{(5-1)^2}+\frac{1}{((s+2)-1)^2}}(r-2) + \sqrt{\frac{1}{((s-1)-1)^2}+\frac{1}{((2s-4)-1)^2}}(rs(r-1) - 2(r+1))/2$$

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Vol: 41 Issue: 11-2022
DOI 10.17605/OSF.IO/U9P3B

$$RBSO_1(G) = \frac{5}{6} + \frac{2}{3}\sqrt{1 + \frac{9}{(s-2)^2}} + \frac{1}{4}\sqrt{2}(r-2) + \frac{1}{2}\sqrt{1 + \frac{16}{(s-1)^2}}(r-2) + \sqrt{2}\sqrt{\frac{1}{(s-2)^2}}(\frac{1}{2}rs(r-1) - r - 1)$$

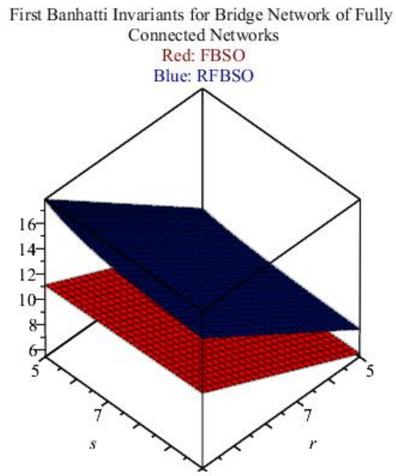


Figure 8: $BSO_1$ and R $BSO_1$ for Gr (Ks, v) over Ks

Fig. 8 shows the results Eq. (1) & (2) of BSO1 indices and its reduced form in red and blue colors respectively in the 3D version.

### 4.3.4 Investigation of Bridge Graphs by $BSO_2$ Indices and its different forms

Proof

$$BSO_2(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\frac{1}{d_u^2} + \frac{1}{d_v^2}}}$$

$$BSO_2(G) = \frac{1}{\sqrt{\frac{1}{4^2}+\frac{1}{7^2}}}(2) + \frac{1}{\sqrt{\frac{1}{4^2}+\frac{1}{(s+1)^2}}}(2) + \frac{1}{\sqrt{\frac{1}{5^2}+\frac{1}{8^2}}}(r-2) + \frac{1}{\sqrt{\frac{1}{5^2}+\frac{1}{(s+2)^2}}}(r-2) + \frac{1}{\sqrt{\frac{1}{(s-1)^2}+\frac{1}{(2s-4)^2}}}(rs(r-1) - 2(r+1))/2$$

$$BSO_2(G) = \frac{40}{41}\sqrt{41} + \frac{8}{\sqrt{1+\frac{16}{(s-1)^2}}} + \frac{5}{2}\sqrt{2}(r-2) + \frac{5}{\sqrt{1+\frac{25}{(s-1)^2}}}(r-2) + \frac{1}{4\sqrt{\frac{1}{(s-1)^2}}}\sqrt{2}\,(\frac{1}{2}rs(r-1) - 2r - 2)$$

$$RBSO_2\ (G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\frac{1}{(d_u-1)^2}+\frac{1}{(d_v-1)^2}}}$$

$$RBSO_2\ (G) = \frac{1}{\sqrt{\frac{1}{(4-1)^2}+\frac{1}{(7-1)^2}}}(2) + \frac{1}{\sqrt{\frac{1}{(4-1)^2}+\frac{1}{((s+1)-1)^2}}}(2) + \frac{1}{\sqrt{\frac{1}{(5-1)^2}+\frac{1}{(8-1)^2}}}(r-2) + \frac{1}{\sqrt{\frac{1}{(5-1)^2}+\frac{1}{((s+2)-1)^2}}}(r-2) + \frac{1}{\sqrt{\frac{1}{((s-1)-1)^2}+\frac{1}{((2s-4)-1)^2}}}(rs(r-1) - 2(r+1))/2$$
2

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
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Vol: 41 Issue: 11-2022
DOI 10.17605/OSF.IO/U9P3B

$$RBSO_2\ (G) = \frac{24}{5} + \frac{6}{\sqrt{1+\frac{9}{(s-2)^2}}} + 2\sqrt{2}(r-2) + \frac{4}{\sqrt{1+\frac{16}{(s-2)^2}}}(r-2) + \frac{1}{4\sqrt{\frac{1}{(s-1)^2}}}\sqrt{2}\ \left(\frac{1}{2}rs(r-1) - 2r - 2\right)$$

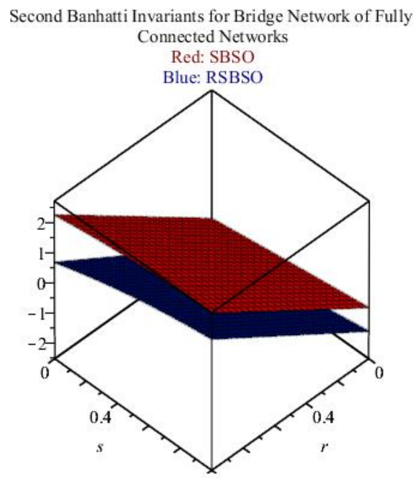


Figure 9: $BSO_2$ and R $BSO_2$ for Gr (Ks, v) over Ks

Fig. 9 shows the results Eq. (3) & (4) of BSO2 indices and their reduced form in red and blue colors respectively in the 3D version.

## 5. CONCLUSIONS

Bridge networks are vastly used in different fields of sciences. On the other hand, TIs have lots of uses and implementations in many fields of computer science, chemistry, biology, informatics, arithmetic, material sciences, and robotics, etc. But the utmost significant application is in the non-exact QSPR and QSAR. TIs are associated with the structure of networks, the backbone of the internet, local area networks and chemical structure. But the present article discusses the First and Second Banhatti Sombor invariants with their reduced form which are freshly presented. These have numerous prediction qualities for different variants of bridge graphs or networks, i.e. Gr (Ps, v), Gr (Cs, v) and Gr (Ks, v). Fig. 2, 5, and 8 give the graphical representation of BSO1 invariants and Fig. 3, 6, and 9 give the graphical representation of BSO2 invariants for the above-mentioned bridge networks. These deduced results will be useful for the modeling of computer networks, the backbone of the internet, memory interconnection networks, power generation networks, chemical structures, image processing, bioinformatics, memory interconnection networks and assembly of robotics,

**AUTHOR CONTRIBUTIONS:**

K. Hamid and M. W. Iqbal; methodology, K. Hamid; software, M. W. Iqbal; validation, K. Hamid and M. W. Iqbal and S. A. Hassan; formal analysis, K. Hamid and H. A. B. Muhammad; investigation, K. Hamid; resources, K. Hamid and M. A. Hamza; data curation, K. Hamid and M. W. Iqbal; writing—original draft preparation, K. Hamid and M. W. Iqbal; writing—review and editing, K. Hamid and S. U. Bhatti; visualization, M. W. Iqbal and Atif; supervision. All authors have read and agreed to the published version of the manuscript.

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