Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)
ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 43 Issue: 12-2024
DOI: 10.5281/zenodo.14356898

ON SOME DIVISOR CORDIAL LABELINGS IN H - GRAPH

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Abstract

In 2011 [11], Varatharajan.et.al., introduced the concept of divisor cordial labeling. Let $G = (\delta(G), \beta(G))$ be a graph with p vertices and q edges. A bijective function S : δ (G) → {1,2,3,….,p}is said to be a divisor cordial labeling, if an induced function S*(bc) $=\begin{cases} 1 & (s(b)|s(c))\ or (s(c)|s(b)) \\ 0 & otherwise \end{cases}$, $\forall$ bc ∈ β(G) satisfies the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \le 1$. A graph which admits a divisor cordial labeling is called a divisor cordial graph. In this paper we investigate the existence of some divisor cordial labelings of the H graph.

Keywords: Graph Labeling, H-Graph, Triplicate Graph, Divisor Cordial Labeling.

1. INTRODUCTION

Rosa introduced the concept of graph labeling in 1967 [9]. Assigning an integers to the edges or vertices or both to the certain conditions is known as graph labeling. Bala and Thirusangu was introduced the concept of the extended triplicate graph of a path P_p in 2011[1]. In 2023[2], the concept of Extended triplicate graph of star ETG($k_{1,p}$) was introduced by Bala.et.al.,.

In 2016[6], the concept of Sum divisor cordial labeling was introduced by Lourdusamy.et.al.,. Let $G = (\delta(G), \beta(G))$ be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \dots\dots, p\}$ is said to be a sum divisor cordial labeling, if an induced function S^*: β(G) → $\{0,1\}$ defined by $S^*(bc) = \begin{cases} 1 & ; \quad if(2|(s(b)+s(c)) \\ 0 & ; \quad otherwise \end{cases}$, $\forall$ bc ∈ β(G) satisfies the condition $|\beta_{S^*}(0) - \beta_{S^*}(1)| \le 1$. A graph which admits a Sum divisor cordial labeling is called as Sum divisor cordial graph.

Gondalia.et.al., was introduced the concept of Subtract divisor cordial labeling in 2019[5]. Let $G = \big(\delta(G), \beta(G)\big)$ be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \dots\dots, p\}$ is said to be a subtract divisor cordial labeling, if an induced function S^*: β (G) → $\{0,1\}$ defined by $S^*(bc) = \begin{cases} 1 & ; \ if\ (2|(s(b) - s(c)) \\ 0 & ; \quad otherwise \end{cases}$, $\forall$ bc ∈ β(G) satisfies the condition $|\beta_{S^*}(0) - \beta_{S^*}(1)| \le 1$. A graph which admits a Subtract divisor cordial labeling is called as Subtract divisor cordial graph.

In 2023[8], the concept of Sum of power p divisor cordial labeling was established by Preetha Lal and Jaslin Melbha. Let $G = \big(\delta(G), \beta(G)\big)$ be a simple graph with p vertices

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and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \ldots\ldots, p\}$ is said to be a sum of power p divisor cordial labeling, if an injective function S^*: β (G) → $\{0,1\}$ defined by $S^*(bc) = \begin{cases} 1\,; if\ 2|(s(b)+s(c))^p \\ 0\ ; \quad otherwise \end{cases}$, $\forall$ bc $\in$ β(G) satisfies the condition $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$. A graph which admits a Sum of power p divisor cordial labeling is called as Sum of power p divisor cordial graph.

In 2020[10], the concept of Modulo divisor cordial labeling was discussed by Shanthini. Let $G = (\delta(G), \beta(G))$ be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \ldots\ldots, p\}$ is said to be a Modulo divisor cordial labeling, if an injective function S^*: β (G) → $\{0,1\}$ defined by $S^*(bc) = \left\lfloor \frac{s(b)}{s(c)} \right\rfloor (\mathrm{mod}2), \mathrm{s(b)} > \mathrm{s(c)}, \forall\ \mathrm{bc} \in$ β(G) satisfies the condition $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$. A graph which admits a Modulo divisor cordial labeling is called as Modulo divisor cordial graph.

In 2012[1], Bala.et.al., introduced the concept of triplicate graph of a path p_p. Let G be a path graph with p vertices and q edges. Let $\delta'(G) = \{b_1, b_2, b_3, \ldots\ldots, b_{p+1}\}$ and $\beta'(G) = \{c_1, c_2, c_3, \ldots\ldots, c_p\}$ be the vertex and edge set of a path p_p. For every $b_i \in \delta'(G)$,construct an ordered triple $\{b_i, b_i', b_i''; 1 \leq i \leq p+1\}$ and for every edge $b_i b_j \in \beta'(G)$, construct four edges $b_i b_j', b_j' b_i'', b_j b_i'\ and\ b_i' b_j''$ where $j = i + 1$, then the graph with this vertex set and edge set is called as Triplicate graph of p_p. It is denoted as $TG(p_p)$. Clearly, the triplicate graph $TG(p_p)$ is disconnected. Let $\delta(G) = \{b_1, b_2, b_3, \ldots\ldots, b_{3p+1}\}$ and $\beta(G) = \{c_1, c_2, c_3, \ldots\ldots, c_{4p}\}$ be the vertex set and edge set of $TG(p_p)$. If p is odd, include a new edge $\{b_{p+1}, b_1\}$ and if p is even, include a new edge $\{b_p, b_1\}$ in the edge set of $TG(p_p)$. This graph is called the Extended triplicate graph of the path p_p and it is denoted by $ETG(p_p)$.

By the interest of the above studies, we investigate the existence of Sum divisor cordial labeling, Subtract divisor cordial labeling, Sum of power p divisor cordial labeling in the context of Triplicate graph of H_p for $p \equiv 0(mod2)$ and Extended triplicate graph of H_p for $p \equiv 1(mod2)$ and also we investigate the existence of Modulo divisor cordial labeling in the context of Extended triplicate graph of H_p for $p \equiv 1(mod2)$.

2. MAIN RESULT

In this section, we provide the structure of Triplicate graph of H_p for $p \equiv 0(mod2)$ and Extended triplicate graph of H_p for $p \equiv 1(mod2)$ and investigate the existence of Sum divisor cordial labeling, Subtract divisor cordial labeling, Sum of power p divisor cordial labeling and Modulo divisor cordial labeling.

2.1 STRUCTURE OF TRIPLICATE OF H_p- GRAPH AND EXTENDED TRIPLICATE OF H_p- GRAPH

Let G be a simple H_p– graph. The triplicate of H_p- graph has the vertex set $\delta(G)$ and edge set $\beta(G)$. The vertex set and edge set are

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For $\boldsymbol{p \equiv 0(mod2)}$

$\delta(G) = \{b_i \cup b_i' \cup b_i'' \cup c_i \cup c_i' \cup c_i'' \,/ 1 \le i \le p\}$ and

$\beta(G) = \{b_i b_{i+1}' \cup b_i'' b_{i+1}' \cup b_i' b_{i+1}'' \cup b_i' b_{i+1} \cup c_i' c_{i+1} \cup c_i' c_{i+1}'' \cup c_i c_{i+1}' \cup c_i'' c_{i+1}' \cup b_{\frac{p}{2}}' c_{\frac{p}{2}+1}'' \cup b_{\frac{p}{2}}' c_{\frac{p}{2}+1} \cup b_{\frac{p}{2}}'' c_{\frac{p}{2}+1}' \cup b_{\frac{p}{2}} c_{\frac{p}{2}+1}' / 1 \le i \le p-1\}$.

Thus, TG(H_p) has $6p$ vertices and $(8p-4)$ edges.

For $\boldsymbol{p \equiv 1(mod2)}$

$\delta(G) = \{b_i \cup b_i' \cup b_i'' \cup c_i \cup c_i' \cup c_i'' \,/ 1 \le i \le p\}$ and

$\beta(G) = \{b_i b_{i+1}' \cup b_i'' b_{i+1}' \cup b_i' b_{i+1}'' \cup b_i' b_{i+1} \cup c_i' c_{i+1} \cup c_i' c_{i+1}'' \cup c_i c_{i+1}' \cup c_i'' c_{i+1}' \cup c_{\frac{p+1}{2}} b_{\frac{p+1}{2}}' \cup b_{\frac{p+1}{2}}' c_{\frac{p+1}{2}}'' \cup b_{\frac{p+1}{2}}'' c_{\frac{p+1}{2}}' \cup b_{\frac{p+1}{2}} c_{\frac{p+1}{2}}' / 1 \le i \le p\}$.

TG(H_p) has 6p vertices and $(8p-4)$ edges. Clearly, TG(H_p) is a disconnected graph . To make this as a connected graph including a new edge $b_1 c_1$ to the edge set $\beta(G)$. The obtained graph has $6p$ vertices and $(8p-3)$ edges is connected and known as Extended triplicate graph of H_p and is denoted by ETG(H_p).

THEOREM 2.1: Triplicate graph of H_p, $p \ge 2$ is a Sum divisor cordial graph for $p \equiv 0(mod2)$.

PROOF: Triplicate graph of H_p, $p \ge 2$ and for $p \equiv 0(mod2)$ has $6p$ vertices and $(8p-4)$ edges.

To show that : TG(H_p), $p \ge 2$ and for $p \equiv 0(mod2)$ is a sum divisor cordial graph

Define a bijective function $S: \delta(G) \to \{1, 2, 3, \dots, 6p\}$ to label the vertices as follows.

For, $1 \le i \le p$	
$s(b_i) = 2i$	$s(c_i) = 2(i$+2p)-1
$s(b_i') = 2(i+p)$	$s(c_i') = 2(i+p)-1$
$s(b_i'') = 2i-1$	$s(c_i'') = 2(i+2p)$

Define an induced function $S^*: \beta(G) \to \{0, 1\}$ such that $S^*(bc) = \begin{cases} 1 & ; \quad if(2|(s(b)+s(c)) \\ 0 & ; \quad otherwise \end{cases}$, $\forall$ bc $\in \beta(G)$ to label the edges as follows.

$S^*\left(b_{\frac{p}{2}}'' c_{\frac{p}{2}+1}'\right) = S^*\left(b_{\frac{p}{2}}' c_{\frac{p}{2}+1}''\right) = 1$
$S^*\left(c_{\frac{p}{2}+1} b_{\frac{p}{2}}'\right) = S^*\left(c_{\frac{p}{2}+1}' b_{\frac{p}{2}}\right) = 0$
For, $1 \le i \le p-1$
$S^*(b_i b_{i+1}') = S^*(b_i' b_{i+1}) = S^*(c_i' c_{i+1}) = S^*(c_{i+1}' c_i) = 1$
$S^*(b_i'' b_{i+1}') = S^*(b_i' b_{i+1}'') = S^*(c_i' c_{i+1}'') = S^*(c_i'' c_{i+1}') = 0$

we get, $\beta_{S^*}(0) = \beta_{S^*}(1) = 4p-2$

Thus, $|\beta_{S^*}(0) - \beta_{S^*}(1)| = |(4p-2) - (4p-2)| \le 1$

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It is clear that the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \leq 1$ is satisfied.

Hence, Triplicate graph of H_p, $p \geq 2$ is Sum divisor cordial graph for $p \equiv 0(mod2)$.

EXAMPLE 2.1: TG(H_4) and its Sum divisor cordial labeling is shown in figure 1.

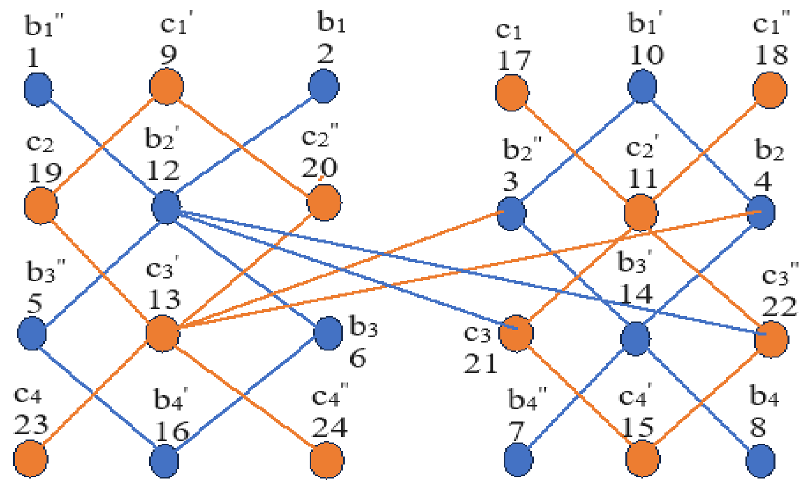

Figure 1

THEOREM 2.2: Extended triplicate graph of H_p, $p > 1$ is a Sum divisor cordial graph for $p \equiv 1(mod2)$.

PROOF: Extended triplicate graph of H_p, $p > 1$and for $p \equiv 1(mod2)$ has $6p$ vertices and $(8p - 3)$ edges.

To show that : ETG(H_p), $p > 1$ and for $p \equiv 1(mod2)$ is a sum divisor cordial graph.

Define a bijective function S: $\delta(G) \rightarrow \{1, 2, 3, \dots, 6p\}$ to label the vertices as follows.

For, $1 \leq i \leq p$	
$s(b_i) = 2(i + 2p)$	$s(c_i) = 2(i + p)$
$s(b_i') = 2(i + 2p) - 1$	$s(c_i') = 2i$
$s(b_i'') = 2i - 1$	$s(c_i'') = 2(i + p) - 1$

Define an induced function $S^*: \beta(G) \rightarrow \{0, 1\}$ such that $S^*(bc) = \begin{cases} 1 & ; \ if(2|(s(b) + s(c)) \\ 0 & ; \ otherwise \end{cases}$, $\forall$ bc $\in \beta(G)$ to label the edges as follows.

$S^*(b_1 c_1) = S^*\left(c'_{\frac{p+1}{2}} b_{\frac{p+1}{2}}\right) = S^*\left(b'_{\frac{p+1}{2}} c''_{\frac{p+1}{2}}\right) = 1$
$S^*\left(c_{\frac{p+1}{2}} b'_{\frac{p+1}{2}}\right) = S^*\left(b''_{\frac{p+1}{2}} c'_{\frac{p+1}{2}}\right) = 0$
For, $1 \leq i \leq p - 1$
$S^*(b_i b'_{i+1}) = S^*(b'_i b_{i+1}) = S^*(c'_i c''_{i+1}) = S^*(c''_i c'_{i+1}) = 0$
$S^*(b''_i b'_{i+1}) = S^*(b'_i b''_{i+1}) = S^*(c'_i c_{i+1}) = S^*(c'_{i+1} c_i) = 1$

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we get, $\beta_{s^*}(0) = 4p - 2$ and $\beta_{s^*}(1) = 4p - 1$

Thus, $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p-2) - (4p-1)| \leq 1$

It is clear that the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \leq 1$ is satisfied.

Hence, Extended triplicate graph of H_p, , $p > 1$ is Sum divisor cordial graph for $p \equiv 1(mod2)$

EXAMPLE 2.2: ETG(H_3) and its Sum divisor cordial labeling is shown in figure 2.

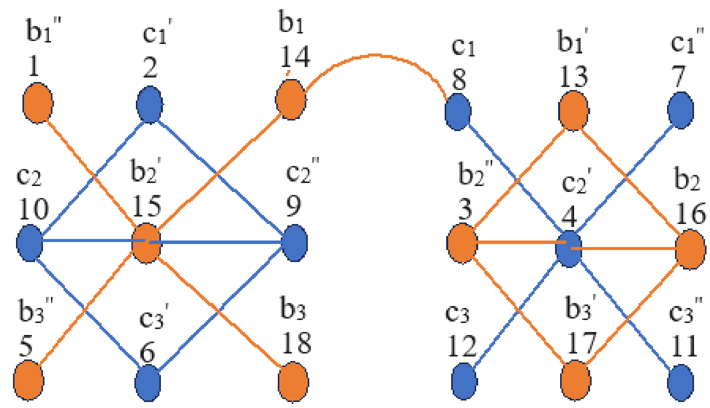

Figure 2

THEOREM 2.3: Triplicate graph of H_p, $p \geq 2$ is a Subtract divisor cordial graph for

$p \equiv 0(mod2)$.

PROOF: Triplicate graph of H_p, $p \geq 2$ and for $p \equiv 0(mod2)$ has $6p$ vertices and $(8p-4)$ edges.

To show that: TG(H_p), $p \geq 2$ and for $p \equiv 0(mod2)$ is as subtract divisor cordial graph.

Define a bijective function $S: \delta(G) \rightarrow \{1, 2, 3, \ldots, 6p\}$ to label the vertices as follows.

For, $1 \leq i \leq p$	
$s(b_i) = 2i$	$s(c_i) = 2(i+2p)-1$
$s(b_i') = 2(i+p)$	$s(c_i') = 2(i+p) - 1$
$s(b_i'') = 2i - 1$	$s(c_i'') = 2(i+2p)$

Define an induced function $S^*: \beta(G) \rightarrow \{0, 1\}$ such that

$S^*(bc) = \begin{cases} 1 \; ; \; if \; (2|s(b) - s(c)) \\ 0 \; ; \quad otherwise \end{cases}$, $\forall$ bc $\in \beta(G)$ to label the edges as follows.

$S^*\left(b''_{\frac{p}{2}} c'_{\frac{p}{2}+1}\right) = S^*\left(b'_{\frac{p}{2}} c''_{\frac{p}{2}+1}\right) = 1$
$S^*\left(c_{\frac{p}{2}+1} b'_{\frac{p}{2}}\right) = S^*\left(c'_{\frac{p}{2}+1} b_{\frac{p}{2}}\right) = 0$

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For, $1 \le i \le p-1$
$S^*(b_ib'_{i+1}) = S^*(b'_ib_{i+1}) = S^*(c'_ic_{i+1}) = S^*(c'_{i+1}c_i) = 1$
$S^*(b''_ib'_{i+1}) = S^*(b'_ib''_{i+1}) = S^*(c'_ic''_{i+1}) = S^*(c''_ic'_{i+1}) = 0$

we get, $\beta_{s^*}(0) = \beta_{s^*}(1) = 4p-2$

Thus, $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p-2)-(4p-2)| \le 1$

It is clear that the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \le 1$ is satisfied.

Hence, Triplicate graph of H_p, $p \ge 2$ is Subtract divisor cordial graph for $p \equiv 0(mod2)$.

EXAMPLE 2.3: TG(H_4) and its Subtract divisor cordial labeling is shown in figure 3.

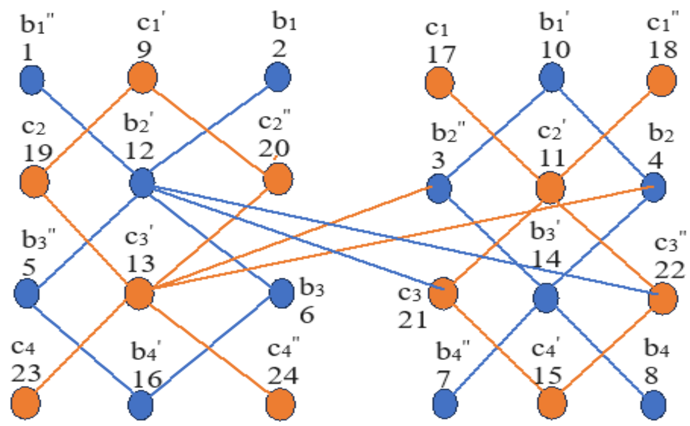

Figure 3

THEOREM 2.4: Extended triplicate graph of H_p, $p > 1$ is a Subtract divisor cordial graph for $p \equiv 1(mod2)$.

PROOF: Extended triplicate graph of H_p, $p > 1$ and for $p \equiv 1(mod2)$ has $6p$ vertices and $(8p-3)$ edges.

To show that : ETG(H_p), $p > 1$and for $p \equiv 1(mod2)$ is a subtract divisor cordial graph.

Define a bijective function $S: \delta(G) \to \{1, 2, 3, \dots, 6p\}$ to label the vertices as follows.

For, $1 \le i \le p$	
$s(b_i) = 2(i+2p)$	$s(c_i) = 2(i+p)$
$s(b'_i) = 2(i+2p)-1$	$s(c'_i) = 2i$
$s(b''_i) = 2i-1$	$s(c''_i) = 2(i+p)-1$

Define an induced function $S^*: \beta(G) \to \{0, 1\}$ such that $S^*(bc) = \begin{cases} 1 \;; & if\ (2|s(b)-s(c)) \\ 0 \;; & otherwise \end{cases}$, $\forall$ bc $\in$ β(G)to label the edges as follows.

$S^*(b_1c_1) = S^*\left(c'_{\frac{p+1}{2}}b_{\frac{p+1}{2}}\right) = S^*\left(b'_{\frac{p+1}{2}}c''_{\frac{p+1}{2}}\right) = 1$
$S^*\left(c_{\frac{p+1}{2}}b'_{\frac{p+1}{2}}\right) = S^*\left(b''_{\frac{p+1}{2}}c'_{\frac{p+1}{2}}\right) = 0$

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For, $1 \le i \le p-1$
$S^*(b_i b'_{i+1}) = S^*(b'_i b_{i+1}) = S^*(c'_i c''_{i+1}) = S^*(c''_i c'_{i+1}) = 0$
$S^*(b''_i b'_{i+1}) = S^*(b'_i b''_{i+1}) = S^*(c'_i c_{i+1}) = S^*(c'_{i+1} c_i) = 1$

we get, $\beta_{s^*}(0) = 4p - 2$ and $\beta_{s^*}(1) = 4p - 1$

Thus, $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p-2) - (4p-1)| \le 1$

It is clear that the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \le 1$ is satisfied.

Hence, Extended triplicate graph of H_p , $p > 1$ is Subtract divisor cordial graph for $p \equiv 1(mod2)$.

EXAMPLE 2.4: ETG(H_3) and its Subtract divisor cordial labeling is shown in figure 4.

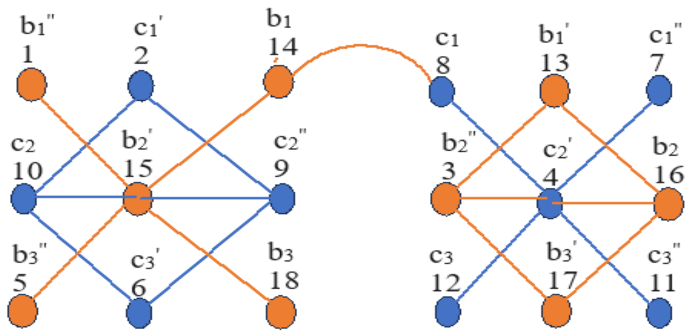

Figure 4

THEOREM 2.5: Triplicate graph of $H_p, p \ge 2$ is a Sum of power p divisor cordial graph for $p \equiv 0(mod2)$.

PROOF: Triplicate graph of H_p, $p \ge 2$ and for $p \equiv 0(mod2)$ has $6p$ vertices and $(8p-4)$ edges.

To show that : TG(H_p), $p \ge 2$ and for $p \equiv 0(mod2)$ is a subtract divisor cordial graph.

Define a bijective function $S: \delta(G) \to \{1, 2, 3, \dots, 6p\}$ to label the vertices as follows.

For, $1 \le i \le p$	
$s(b_i) = 2(i+2p)$	$s(c_i) = 2i$
$s(b'_i) = 2(i+p)$	$s(c'_i) = 2(i+p) - 1$
$s(b''_i) = 2i - 1$	$s(c''_i) = 2(i+2p) - 1$

Define an induced function $S^*: \beta(G) \to \{0, 1\}$ such that

$S^*(bc) = \begin{cases} 1 \;\; ; \;\; if \;\; (2|\,(s(b)+s(c))^p\,) \\ 0 \;\; ; \qquad\qquad otherwise \end{cases}$, $\forall$ bc $\in \beta(G)$ to label the edges as follows.

$S^*\left(b''_{\frac{p}{2}} c'_{\frac{p}{2}+1}\right) = S^*\left(c_{\frac{p}{2}+1} b'_{\frac{p}{2}}\right) = 1$
$S^*\left(b'_{\frac{p}{2}} c''_{\frac{p}{2}+1}\right) = S^*\left(c'_{\frac{p}{2}+1} b_{\frac{p}{2}}\right) = 0$

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For, $1 \le i \le p-1$
$S^*(b_i b'_{i+1}) = S^*(b'_i b_{i+1}) = S^*(c'_i c''_{i+1}) = S^*(c''_i c'_{i+1}) = 1$
$S^*(b''_i b'_{i+1}) = S^*(b'_i b''_{i+1}) = S^*(c'_i c_{i+1}) = S^*(c'_{i+1} c_i) = 0$

we get, $\beta_{s^*}(0) = \beta_{s^*}(1) = 4p-2$

Thus, $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p-2)-(4p-2)| \le 1$

It is clear that the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \le 1$ is satisfied.

Hence, Triplicate graph of H_p, $p \ge 2$ is Sum of power p divisor cordial graph for $p \equiv 0(mod2)$.

EXAMPLE 2.5: TG(H_4) and its Sum of power p divisor cordial labeling is shown in figure 5.

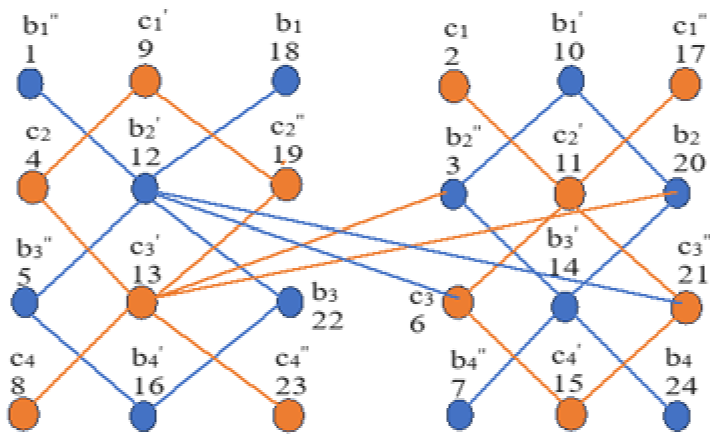

Figure 5

THEOREM 2.6: Extended triplicate graph of $H_p, p > 1$ is a Sum of power p divisor cordial graph for $p \equiv 1(mod2)$.

PROOF: Extended triplicate graph of H_p, $p > 1$ and for $p \equiv 1(mod2)$ has $6p$ vertices and $(8p-3)$ edges.

To show that : ETG(H_p), $p > 1$ and for $p \equiv 1(mod2)$ is a sum of power p divisor cordial graph.

Define a bijective function $S: \delta(G) \to \{1, 2, 3, \dots, 6p\}$ to label the vertices as follows.

For, $1 \le i \le p$	
$s(b_i) = 2(i+p)$	$s(c_i) = 2(2p+i)$
$s(b'_i) = 2(i+p) - 1$	$s(c'_i) = 2i$
$s(b''_i) = 2i - 1$	$s(c''_i) = 2(i+2p) - 1$

Define an induced function $S^*: \beta(G) \to \{0, 1\}$ such that

$S^*(bc) = \begin{cases} 1 \ ; \ if\ (2 |\ (s(b)+s(c))^p\ ) \\ 0 \ ; \quad otherwise \end{cases}$, $\forall$ bc $\in \beta(G)$ to label the edges as follows.

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Vol: 43 Issue: 12-2024
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$S^*(b_1c_1) = S^*\left(c'_{\frac{p+1}{2}}b_{\frac{p+1}{2}}\right) = S^*\left(b'_{\frac{p+1}{2}}c''_{\frac{p+1}{2}}\right) = 1$
$S^*\left(c_{\frac{p+1}{2}}b'_{\frac{p+1}{2}}\right) = S^*\left(b''_{\frac{p+1}{2}}c'_{\frac{p+1}{2}}\right) = 0$
For, $1 \le i \le p-1$
$S^*(b_ib'_{i+1}) = S^*(b'_ib_{i+1}) = S^*(c'_ic''_{i+1}) = S^*(c''_ic'_{i+1}) = 0$
$S^*(b''_ib'_{i+1}) = S^*(b'_ib''_{i+1}) = S^*(c'_ic_{i+1}) = S^*(c'_{i+1}c_i) = 1$

we get, $\beta_{s^*}(0) = 4p-1$ and $\beta_{s^*}(1) = 4p-2$

Thus, $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p-1)-(4p-2)| \le 1$

It is clear that the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \le 1$ is satisfied.

Hence, Extended triplicate graph of H_p , $p > 1$ is Sum of power n divisor cordial graph for $p \equiv 1(mod2)$.

EXAMPLE 2.6: ETG(H_5) and its Sum of power p divisor cordial labeling is shown in figure 6.

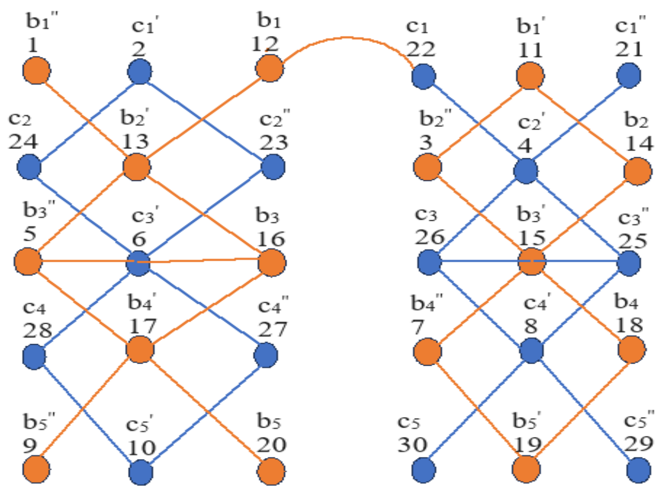

Figure 6

THEOREM 2.7: Extended triplicate graph of H_p, $p > 1$ is a Modulo divisor cordial graph for $p \equiv 1(mod2)$.

PROOF: Extended triplicate graph of H_p , $p > 1$ and for $p \equiv 1(mod2)$ has $6p$ vertices and $(8p-3)$ edges.

To show that: ETG(H_p), $p > 1$ and for $p \equiv 1(mod2)$ is a modulo divisor cordial graph.

Define a bijective function $S: \delta(G) \to \{1, 2, 3, \dots, 6p\}$ to label the vertices as follows.

For, $1 \le i \le p$	
$s(b_i) = 2p + i$	$s(c_i) = 3p + i$
$s(b'_i) = 4p + i$	$s(c'_i) = p + i$
$s(b''_i) = i$	$s(c''_1) = p,\ s(c''_p) = 5p + 1$
For, $2 \le i \le p-1$	$s(c''_i) = 5p + i$

Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)
ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 43 Issue: 12-2024
DOI: 10.5281/zenodo.14356898

Define an induced function $S^{*}:\beta(G)\rightarrow\{0,1\}$ such that

$S^{*}(bc)=\left\lfloor\frac{s(b)}{s(c)}\right\rfloor(\mathrm{mod}2), \mathrm{s(b)}>\mathrm{s(c)}, \forall\ \mathrm{bc}\in\beta(\mathrm{G})$ to label the edges.

we get,

For, $p\neq 5$	$\beta_{s^*}(0)=4p-2$	$\beta_{s^*}(1)=4p-1$	$\lvert\beta_{s^*}(0)-\beta_{s^*}(1)\rvert=\lvert(4p-2)-(4p-1)\rvert\leq 1$
For, $p=5$	$\beta_{s^*}(0)=4p-1$	$\beta_{s^*}(1)=4p-2$	$\lvert\beta_{s^*}(0)-\beta_{s^*}(1)\rvert=\lvert(4p-1)-(4p-2)\rvert\leq 1$

From the above two cases, It is clear that the condition $|\beta_{s^*}(0)-\beta_{s^*}(1)|\leq 1$ is satisfied.

Hence, Extended triplicate graph of $H_p, p>1$ is Modulo divisor cordial graph for $p\equiv 1(mod2)$.

EXAMPLE 2.7: ETG(H_3) and ETG(H_5), its Modulo divisor cordial labeling is shown in figure 7 and figure 8.

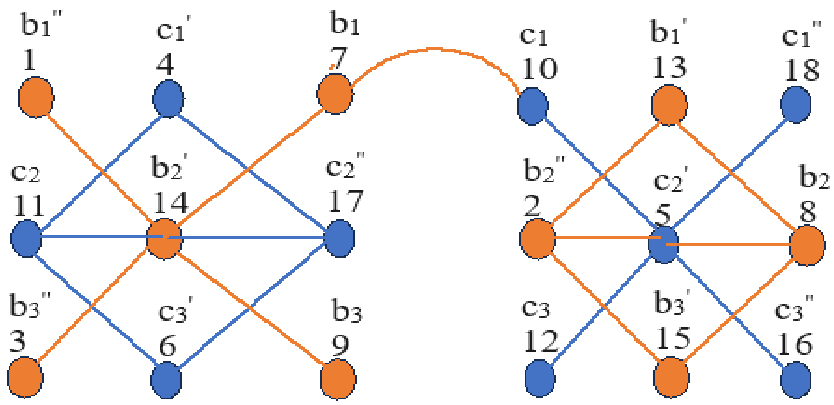

Figure 7

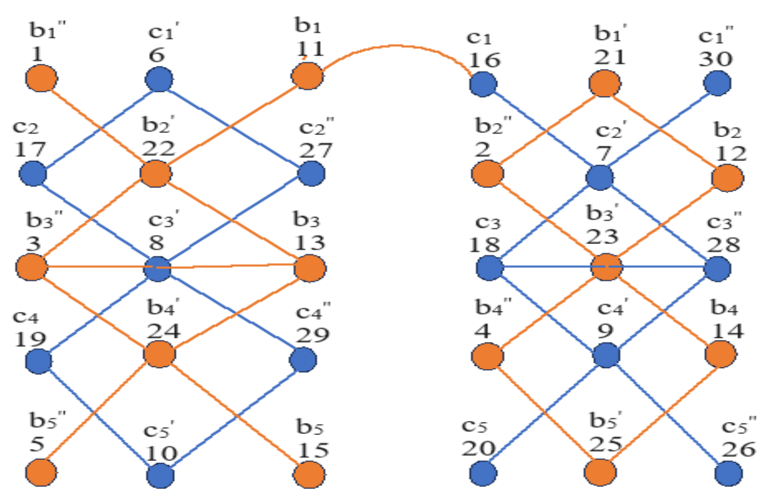

Figure 8

ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 43 Issue: 12-2024
DOI: 10.5281/zenodo.14356898

CONCLUSION

In this paper, we have investigated the existence of Sum divisor cordial labeling, Subtract divisor cordial labeling, Sum of power p divisor cordial labeling in Triplicate graph of H_p for $p \equiv 0(mod2)$ and Extended triplicate graph of H_p for $p \equiv 1(mod2)$ and also we investigate the existence of Modulo divisor cordial labeling in Extended triplicate graph of H_p for $p \equiv 1(mod2)$.

Reference

1) Bala.E, Thirusangu.K, Some graph labelings in Extended triplicate graph of a path $p_{n,}$ International review in Applied Engineering research,Vol.1.No(2011),pp.81-92.

2) Bala.S, Saraswathy.S, Thirusangu.K, Some labelings on extended triplicate graph of star, Proceedings of the international conference on recent in application of mathematics 2023, pp.302-305.

3) Bala.S, Saraswathy.S, Thirusangu.K, On some cordial labelings in a family of H-graph, IPE journal of management, 2024, Volume 14, No 18, ISSN 2249-9040,pp 83-96.

4) Cahit.I, Cordial graphs:A Weaker version of graceful and harmonious graph. ARS combinatorial, 1987, Volume 78, pp 179-199.

5) Gondalia.J.T and Rokad.A.H, Subtract Divisor cordial labeling, International Journal of innovative technology and exploring Engineering, Apl 2019, Volume 8,Issue 684, pp.541-545.

6) Lourdusamy.A and Patrick.F, Sum divisor cordial graphs, Proyecciones Journal of math, March2 2016, Volume.35, Issue 1.

7) Murugesan. S,Jayaraman. D and Shiama. J, Square divisor cordial graphs, International Journal of computer applications, 2013, Voluem 64, Issue 22, pp 1-9.

8) Preetha Lal. P and Jaslin Melbha. M, Sum of power n divisor cordial labeling for subdivision graphs, Eur.Chem.Bull., 2023, Volume 12, Issue 8, pp 3038-3050.

9) Rosa.A, on certain valuation of the vertices of graph. Theory of graphs (International symposium, Rome), July 1996, Gordan and Breach.N.Y and Dunad paris(1967),pp.349-335.

10) Shanthini. C, Modulo divisor cordial for path related graphs, Malaya journal of matematik, ISSN 2319-3786, 2020, Volume S, Issue 2, pp 4104 – 4107.

11) Varatharajan.R, Navanaeethakrishnan. S and Nagarajan.K, Divisor cordial Graphs, International Journal of mathematics and combinatorics, 2011, Volume 4, pp.15-25.