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TOPOLOGICAL STUDY OF LINE GRAPH OF SUBDIVISION OF SOME CONVEX POLYTOPES BY NEIGHBORHOOD M-POLYNOMIAL

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Abstract

Investigation of topological descriptors is one of the most active research field in chemical graph theory. It illustrates atomic construction mathematically and is utilized in the advancement of qualitative structure activity/ property relationships. There are several topological indices that have been introduced in theoretical chemistry to measure the properties of molecular topology. Among these tools, M-Polynomial and Neighborhood M-Polynomial are most important. This research work defines the Neighborhood M-polynomial of line graph of subdivision of some convex polytopes to obtain neighborhood degree-based topological indices. For a graph$\mathcal{G}$, the Neighborhood M-polynomial is defined as$NM(\mathcal{G}) = \sum_{i\le j} \Upsilon_{i,j}\, m^i n^j$, where $\Upsilon_{i,j}$, (i, j ≥ 1), is the number of edges uv of $\mathcal{G}$ such that $d_{\mathcal{G}}$ (u) = i and $d_{\mathcal{G}}$ (v) = j. The line graph L ($\mathcal{G}$) of a graph $\mathcal{G}$ is a graph whose vertex set is one-to-one correspondence with the edge set of the graph $\mathcal{G}$ and two vertices of L ($\mathcal{G}$) are adjacent if and only if the corresponding edges are adjacent in$\mathcal{G}$. The subdivision graph S ($\mathcal{G}$) of a graph $\mathcal{G}$ is the graph obtained by inserting a new vertex into each edge of$\mathcal{G}$. From the neighborhood M-polynomial, some neighborhood degree sum based topological indices are recovered. In future, the importance of this research is to uncover fresh findings that will aid in the development of more exact and accurate estimates in the field of QSPR and QSAR.

Keywords: Topological Index, Chemical Graph Theory, Subdivision Graph, M-Polynomial.

1. INTRODUCTION

A graph can be recognized by a numerical number, a polynomial, a combination of numbers or a lattice which speaks to the entire diagram, and these representations are expected to be uniquely characterized for that chart. A graph $\mathcal{G}$ (V; E) is represented by vertices (apex, nodes or points) which are joined by edges. Chemical graph theory (CGT), also known as molecular graph theory, is an interdisciplinary field that uses graph theory to explore molecular structures. A molecular graph is a finite, simple graph in which the vertices correspond to atoms and the edges to molecule bonds. The mathematical tools of graph theory can be utilized to model chemical compounds [5].

In quantum chemistry, graph theory is also particularly useful for investigating the structural features of chemical molecules. The qualities of a chemical molecule can be studied using a numerical measure called topological index (TI). A topological index, also known as a molecular structure descriptor or graph index, is a single integer that

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can be arisen from a molecular graph and used to characterize some attribute of the underlying molecule [12].

The term “graph” was proposed by Sylvester in paper published in 1878. A graph $\mathcal{G}$ (V; E) is represented by vertices (nodes or points) which are connected by edges. Graph theory, now a days applied in various branches of studies. A branch of graph theory that deals with the study of molecular structure is called chemical graph theory or molecular graph. A chemical graph is a finite, simple graph where we ignored hydrogen, and the atoms of molecules are represented by vertices and bonds by the edges. Therefore, Graph theory is used as a tool to understand the structural properties of a chemical compound [5].

Convex polytopes [4] are basic geometric figures. Their theory's beauty is now complemented by its importance in a variety of other mathematical fields, including integration theory, algebraic topology, and algebraic geometry, as well as linear and combinatorial optimization [13].

2. RESULTS AND DISCUSSION

2.1. Convex Polytope T_r

The graph of convex polytope T_r consists of 3-sided faces, 5-sided faces and r-sided face. The convex polytope T_r for r = 8 is shown in Figure 1. We obtain the subdivision S(T_r) by adding additional vertex between each pair of neighboring vertices of T_r. S(T_8) is shown in Figure 2. The S(T_r) consists of 10r vertices out of which 5r vertices are of degree 2, 2r vertices are of degree 3 and r vertices are of degree 4. So, we have |E(S(T_r))| = 10r. The line graph of subdivision of T_r is L(S(T_r)) for r = 8 is shown in Figure 3. The line graph of subdivision of T_r consists of 10r vertices out which 6r vertices are of degree 3 and 4r vertices are of degree 4. So, we have |E(L(S(T_r)))| = 17r.

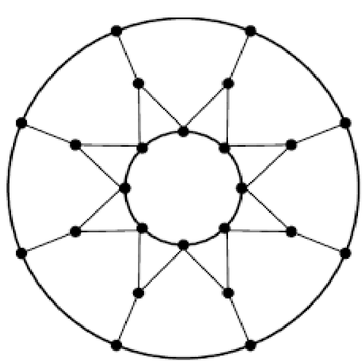

Figure 1: Convex Polytope T_8

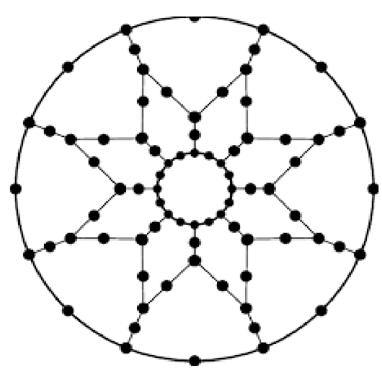

Figure 2: Subdivision of T_8

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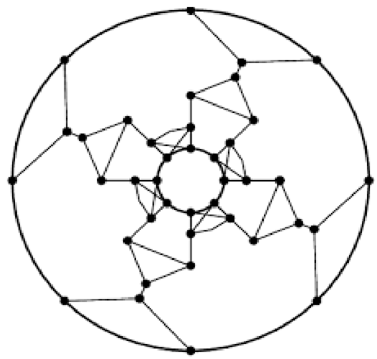

Figure 3: Line graph of subdivision of

Theorem 1.

Let $\mathcal{G}$ be a line graph of subdivision of convex polytope T_r. Then we have,

$$NM(\mathcal{G};m,n) = (5r)m^9n^9 + (2r)m^9n^{10} + (r)m^{10}n^{10} + (2r)m^{10}n^{15} + (r)m^{15}n^{15} + (4r)m^{15}n^{16} + (2r)m^{16}n^{16}$$

Proof:

The line graph of subdivision of convex polytope T_r (L(S (T_r))) has 17r number of edges.

Its edge set can be partitioned as follows:

|{9,9} | = |{$uv \in E(\mathcal{G})$: δu = 9, δv = 9}| = 5r =ϒ(9,9) ,

|{9,10} | = |{$uv \in E(\mathcal{G})$: δu = 9, δv = 10}| = 2r = ϒ (9,10) ,

|{10,10} | = |{$uv \in E(\mathcal{G})$: δu = 10, δv = 10}| = r = ϒ (10,10) ,

|{10,15} | = |{$uv \in E(\mathcal{G})$: δu = 10, δv = 15}| = 2r = ϒ (10,15) ,

|{15,15} | = |{$uv \in E(\mathcal{G})$: δu = 15, δv = 15}| = r = ϒ (15,15) ,

|{15,16} | = |{$uv \in E(\mathcal{G})$: δu = 15, δv = 16}| = 4r = ϒ (15,16) ,

|{16,16} | = |{$uv \in E(\mathcal{G})$: δu = 16, δv = 16}| = 2r = ϒ (16,16).

From the definition, the NM-polynomial of $\mathcal{G}$ is obtained as follows.

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$$NM(\mathcal{G}) = \sum_{i \le j} \Upsilon_{(i,j)}\, m^i n^j$$

$= \gamma_{(9,9)} m^9 n^9 + \gamma_{(9,10)} m^9 n^{10} + \gamma_{(10,10)} m^{10} n^{10} + \gamma_{(10,15)} m^{10} n^{15} + \gamma_{(15,15)} m^{15} n^{15} + \gamma_{(15,16)} m^{15} n^{16} + \gamma_{(16,16)} m^{16} n^{16}$.

$NM(\mathcal{G}) = (5r)m^9n^9 + (2r)m^9n^{10} + (r)m^{10}n^{10} + (2r)m^{10}n^{15} + (r)m^{15}n^{15} + (4r)m^{15}n^{16} + (2r)m^{16}n^{16}$

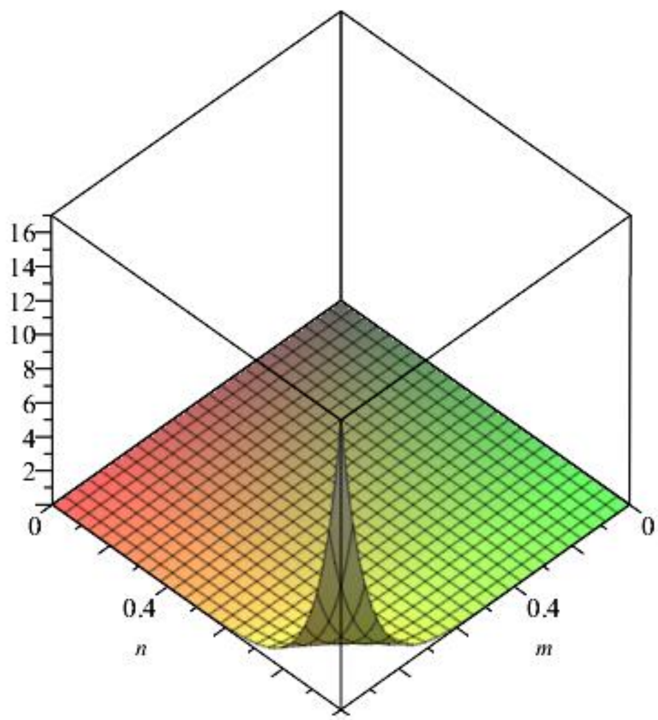

Figure 4: 3D plot of line graph of subdivision of convex polytope T_r

Now using this NM-polynomial, we calculate some Neighborhood M-Polynomial and degree based TIs of line graph of subdivision of convex polytope T_r as follows

Theorem 2.

Let $\mathcal{G}$ be a line graph of subdivision of convex polytope T_r. Then we have,

1. $M_1'(\mathcal{G}) = 416r$,
2. $M_2^*(\mathcal{G}) = 2682r$,
3. $F_N^*(\mathcal{G}) = 5420r$,

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4. $^{nm}M_2(\mathcal{G}) = \frac{7061r}{51840}$,

5. $NR_\alpha(\mathcal{G}) = (2682)^\alpha r$,

6. $ND_3(\mathcal{G}) = 73104r$,

7. $ND_5(\mathcal{G}) = \frac{6187r}{180}$,

8. $NH(\mathcal{G}) = 1.475r$,

9. $NI(\mathcal{G}) = 103.441r$,

10. $S(\mathcal{G}) = 5872.106r$.

Proof:

Let,

$NM(\mathcal{G}; m, n) = (5r)m^9n^9 + (2r)m^9n^{10} + (r)m^{10}n^{10} + (2r)m^{10}n^{15} + (r)m^{15}n^{15} + (4r)m^{15}n^{16} + (2r)m^{16}n^{16}$

………. (A)

Then, 1st neighborhood first Zagreb index is

$$M_1'(\mathcal{G}) = (D_m + D_n)|_{m=n=1}$$

Where,

$$D_m = m\left\{\frac{\partial\left(\mathrm{NM}(\mathcal{G}; \mathrm{m}, \mathrm{n})\right)}{\partial \mathrm{m}}\right\}$$

Applying this on equation (A), we get

$$D_m = m\{9(5r)m^8n^9 + 9(2r)m^8n^{10} + 10(r)m^9n^{10} + 10(2r)m^9n^{15} + 15(r)m^{14}n^{15} + 15(4r)m^{14}n^{16} + 16(2r)m^{15}n^{16}\}$$

$$= 9(5r)m^9n^9 + 9(2r)m^9n^{10} + 10(r)m^{10}n^{10} + 10(2r)m^{10}n^{15} + 15(r)m^{15}n^{15} + 15(4r)m^{15}n^{16} + 16(2r)m^{16}n^{16}$$ …………… (1)

Similarly, following same procedure we can find D_n

$$D_n = n\left\{\frac{\partial\left(\mathrm{NM}(\mathcal{G}; \mathrm{m}, \mathrm{n})\right)}{\partial \mathrm{n}}\right\}$$

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$D_n = n\{9(5r)m^9n^8 + 10(2r)m^9n^9 + 10(r)m^{10}n^9 + 15(2r)m^{10}n^{14} + 15(r)m^{15}n^{14} + 16(4r)m^{15}n^{15} + 16(2r)m^{16}n^{15}\}$

$= 9(5r)m^9n^9 + 10(2r)m^9n^{10} + 10(r)m^{10}n^{10} + 15(2r)m^{10}n^{15} + 15(r)m^{15}n^{15} + 16(4r)m^{15}n^{16} + 16(2r)m^{16}n^{16}$ …………… (2)

Now, by adding eq (1) and (2) we calculate

$D_m + D_n = 18(5r)m^9n^9 + 19(2r)m^9n^{10} + 20(r)m^{10}n^{10} + 25(2r)m^{10}n^{15} + 30(r)m^{15}n^{15} + 31(4r)m^{15}n^{16} + 32(2r)m^{16}n^{16}$ …………… (3)

Now put m=n=1 in eq (3), we get this $(D_m + D_n)|_{m=n=1}$

$= 18(5r) + 19(2r) + 20(r) + 25(2r) + 30(r) + 31(4r) + 32(2r)$

= 90r+38r+20r+50r+30r+124r+64r

$M_1'(\mathcal{G}) = 416r.$

For second neighborhood Zagreb index

$$M_2^*(\mathcal{G}) = (D_mD_n)|_{m=n=1}$$

By multiplying eq (1) and (2), we get

$D_mD_n = 81(5r)m^9n^9 + 90(2r)m^9n^{10} + 100(r)m^{10}n^{10} + 150(2r)m^{10}n^{15} + 225(r)m^{15}n^{15} + 240(4r)m^{15}n^{16} + 256(2r)m^{16}n^{16}$ …………… (4)

By putting m=n=1 we get, $(D_mD_n)|_{m=n=1}$

$= 81(5r) + 90(2r) + 100(r) + 150(2r) + 225(r) + 240(4r) + 256(2r)$

= 405r+180r+100r+300r+225r+960r+512r

$M_2^*(\mathcal{G}) = 2682r.$

For Neighborhood forgotten topological index:

$$F_N^*(\mathcal{G}) = (D_m^2 + D_n^2)|_{m=n=1}$$

Now we take derivative of eq (1), to get

$D_m^2 = 81(5r)m^9n^9 + 81(2r)m^9n^{10} + 100(r)m^{10}n^{10} + 100(2r)m^{10}n^{15} + 225(r)m^{15}n^{15} + 225(4r)m^{15}n^{16} + 256(2r)m^{16}n^{16}$ …………... (5)

Now take derivative of eq (1), to get this

$D_n^2 = 81(5r)m^9n^9 + 100(2r)m^9n^{10} + 100(r)m^{10}n^{10} + 225(2r)m^{10}n^{15} + 225(r)m^{15}n^{15} + 256(4r)m^{15}n^{16} + 256(2r)m^{16}n^{16}$ …………… (6)

Adding eq (5) and (6), to get this $(D_m^2 + D_n^2)|_{m=n=1}$

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$D_m^2 + D_n^2 = 162(5r)m^9n^9 + 181(2r)m^9n^{10} + 200(r)m^{10}n^{10} + 325(2r)m^{10}n^{15} + 450(r)m^{15}n^{15} + 481(4r)m^{15}n^{16} + 512(2r)m^{16}n^{16}$

Put m=n=1in above equation, we get,

$= 162(5r) + 181(2r) + 200(r) + 325(2r) + 450(r) + 481(4r) + 512(2r)$

= 810r+362r+200r+650r+450r+192r+1024r

$F_N^*(\mathcal{G}) = 5420r.$

Neighborhood second modified Zagreb index:

$$^{nm}M_2(\mathcal{G}) = (S_mS_n)|_{m=n=1}$$

As we know,

$$S_m = \int_0^m \frac{f(t,n)}{t} dt$$

Applying this on eq (A), we get

$S_m = \int_0^m \{\frac{(5r)t^9n^9}{t} + \frac{(2r)t^9n^{10}}{t} + \frac{(r)t^{10}n^{10}}{t} + \frac{(2r)t^{10}n^{15}}{t} + \frac{(r)t^{15}n^{15}}{t} + \frac{(4r)t^{15}n^{16}}{t} + \frac{(2r)t^{16}n^{16}}{t}\} dt$

$= \int_0^m \{(5r)t^8n^9 + (2r)t^8n^{10} + (r)t^9n^{10} + (2r)t^9n^{15} + (r)t^{14}n^{15} + (4r)t^{14}n^{16} + (2r)t^{15}n^{16}\}$ dt

$= | \frac{(5r)t^9n^9}{9} + \frac{(2r)t^9n^{10}}{9} + \frac{(r)t^{10}n^{10}}{10} + \frac{(2r)t^{10}n^{15}}{10} + \frac{(r)t^{15}n^{15}}{15} + \frac{(4r)t^{15}n^{16}}{15} + \frac{(2r)t^{16}n^{16}}{16} |_0^m$

$= \frac{(5r)m^9n^9}{9} + \frac{(2r)m^9n^{10}}{9} + \frac{(r)m^{10}n^{10}}{10} + \frac{(2r)m^{10}n^{15}}{10} + \frac{(r)m^{15}n^{15}}{15} + \frac{(4r)m^{15}n^{16}}{15} + \frac{(2r)m^{16}n^{16}}{16}$

…………… (7)

Similarly, following above procedure we get

$S_n = \int_0^n \frac{f(m,t)}{t} dt$

$S_n = \frac{(5r)m^9n^9}{9} + \frac{(2r)m^9n^{10}}{10} + \frac{(r)m^{10}n^{10}}{10} + \frac{(2r)m^{10}n^{15}}{15} + \frac{(r)m^{15}n^{15}}{15} + \frac{(4r)m^{15}n^{16}}{16} + \frac{(2r)m^{16}n^{16}}{16}$

…………… (8)

Now we can multiply eq (7) and (8), to get

$S_mS_n = \frac{(5r)m^9n^9}{81} + \frac{(2r)m^9n^{10}}{90} + \frac{(r)m^{10}n^{10}}{100} + \frac{(2r)m^{10}n^{15}}{150} + \frac{(r)m^{15}n^{15}}{225} + \frac{(4r)m^{15}n^{16}}{240} + \frac{(2r)m^{16}n^{16}}{256}$

…………... (9)

Putting m=n=1 in eq (9), we get $(S_mS_n)|_{m=n=1}$

$= \frac{(5r)}{81} + \frac{(2r)}{90} + \frac{(r)}{100} + \frac{(2r)}{150} + \frac{(r)}{225} + \frac{(4r)}{240} + \frac{(2r)}{256}$

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$^{nm}M_2(\mathcal{G}) = \frac{7061}{51840}r$ Or = 0.1362r.

Neighborhood general Randic index:

$NR_\alpha(\mathcal{G}) = (D_m^\alpha D_n^\alpha)|_{m=n=1}$

Using results from eq (4), we get

$$D_m^\alpha D_n^\alpha = (81)^\alpha(5r)m^9n^9 + (90)^\alpha(2r)m^9n^{10} + (100)^\alpha(r)m^{10}n^{10} + (150)^\alpha(2r)m^{10}n^{15} + (225)^\alpha(r)m^{15}n^{15} + (240)^\alpha(4r)m^{15}n^{16} + (256)^\alpha(2r)m^{16}n^{16}$$

$D_m^\alpha D_n^\alpha = (2682^{)\alpha}$r.

Third ND_e index:

$ND_3(\mathcal{G}) = [D_mD_n(D_m + D_n)]|_{m=n=1}$

By multiplying results of eq (3) and (4), we get that

$$D_mD_n(D_m + D_n) = 1458(5r)m^9n^9 + 1710(2r)m^9n^{10} + 2000(r)m^{10}n^{10} + 3750(2r)m^{10}n^{15} + 6750(r)m^{15}n^{15} + 7440(4r)m^{15}n^{16} + 8192(2r)m^{16}n^{16}$$

Now putting m=n=1, we get this $[D_mD_n(D_m + D_n)]|_{m=n=1}$

$= 1458(5r) + 1710(2r) + 2000(r) + 3750(2r) + 6750(r) + 7440(4r) + 8192(2r)$

= 7290r+3420r+2000r+7500r+6750r+29760r+16384r

$ND_3(\mathcal{G}) = 73104r.$

Fifth ND_e index:

$ND_5(\mathcal{G}) = (D_mS_n + S_mD_n)|_{m=n=1}$

Multiplying eq (1) and (8), we get

$$D_mS_n = \frac{9(5r)m^9n^9}{9} + \frac{9(2r)m^9n^{10}}{10} + \frac{10(r)m^{10}n^{10}}{10} + \frac{10(2r)m^{10}n^{15}}{15} + \frac{15(r)m^{15}n^{15}}{15} + \frac{15(4r)m^{15}n^{16}}{16} + \frac{16(2r)m^{16}n^{16}}{16}$$

$$= \frac{9(5r)m^9n^9}{9} + \frac{9(2r)m^9n^{10}}{10} + \frac{10(r)m^{10}n^{10}}{10} + \frac{10(2r)m^{10}n^{15}}{15} + \frac{15(r)m^{15}n^{15}}{15} + \frac{15(4r)m^{15}n^{16}}{16} + \frac{16(2r)m^{16}n^{16}}{16} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

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Now multiply eq (7) and (2), we get this

$$S_mD_n = \frac{9(5r)m^9n^9}{9} + \frac{10(2r)m^9n^{10}}{9} + \frac{10(r)m^{10}n^{10}}{10} + \frac{15(2r)m^{10}n^{15}}{10} + \frac{15(r)m^{15}n^{15}}{15} + \frac{16(4r)m^{15}n^{16}}{15} + \frac{16(2r)m^{16}n^{16}}{16}$$

$$= (5r)m^9n^9 + \frac{10(2r)m^9n^{10}}{9} + (r)m^{10}n^{10} + \frac{15(2r)m^{10}n^{15}}{10} + (r)m^{15}n^{15} + \frac{16(4r)m^{15}n^{16}}{15} + (2r)m^{16}n^{16} \qquad \ldots\ldots\ldots\ldots\ldots (11)$$

Now, by adding eq (10) and (11) we calculate

$$D_mS_n + S_mD_n = 2(5r)m^9n^9 + \frac{181(2r)m^9n^{10}}{90} + 2(r)m^{10}n^{10} + \frac{13(2r)m^{10}n^{15}}{6} + 2(r)m^{15}n^{15} + \frac{481(4r)m^{15}n^{16}}{240} + 2(2r)m^{16}n^{16} \qquad \ldots\ldots\ldots\ldots\ldots (12)$$

Now putting m=n=1 this will become, $(D_mS_n + S_mD_n)|_{m=n=1}$

$$= 2(5r) + \frac{181(2r)}{90} + 2(r) + \frac{13(2r)}{6} + 2(r) + \frac{481(4r)}{240} + 2(2r)$$

$$= 10r + \frac{181}{45}r + 2r + \frac{13}{3}r + 2r + \frac{481}{60}r + 4r$$

$$ND_5(\mathcal{G}) = \frac{6187}{180}r \quad or \ = 34.372.$$

Neighborhood Harmonic index:

$NH(\mathcal{G}) = 2S_mJ[NM(\mathcal{G})]|_{m=1}$

As we know, $Jf(m,n) = f(m,m)$

Now, we are going to apply J on eq (A).

$$J\big(NM(\mathcal{G})\big) = (5r)m^{18} + (2r)m^{19} + (r)m^{20} + (2r)m^{25} + (r)m^{30} + (4r)m^{31} + (2r)m^{32}$$

$$S_mJ[NM(\mathcal{G})] = \frac{(5r)m^{18}}{18} + \frac{(2r)m^{19}}{19} + \frac{(r)m^{20}}{20} + \frac{(2r)m^{25}}{25} + \frac{(r)m^{30}}{30} + \frac{(4r)m^{31}}{31} + \frac{(2r)m^{32}}{32}$$

$$2S_mJ[NM(\mathcal{G})] = \frac{(5r)m^{18}}{9} + \frac{(4r)m^{19}}{19} + \frac{(r)m^{20}}{10} + \frac{(4r)m^{25}}{25} + \frac{(r)m^{30}}{15} + \frac{(8r)m^{31}}{31} + \frac{(2r)m^{32}}{16}$$

Putting m=1, we get $2S_mJ[NM(\mathcal{G})]|_{m=1}$

$$= \frac{(5r)}{9} + \frac{(4r)}{19} + \frac{(r)}{10} + \frac{(4r)}{25} + \frac{(r)}{15} + \frac{(8r)}{31} + \frac{(2r)}{16}$$

$NH(\mathcal{G}) = 1.475r.$

Neighborhood inverse sum index:

$NI(\mathcal{G}) = [S_mJ(D_mD_n)]|_{m=1}$

Applying J on eq (4)

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$J(D_mD_n) = 81(5r)m^{18} + 90(2r)m^{19} + 100(r)m^{20} + 150(2r)m^{25} + 225(r)m^{30} + 240(4r)m^{31} + 256(2r)m^{32}$

$S_mJ(D_mD_n) = \frac{81(5r)m^{18}}{18} + \frac{90(2r)m^{19}}{19} + \frac{100(r)m^{20}}{20} + \frac{150(2r)m^{25}}{25} + \frac{225(r)m^{30}}{30} + \frac{240(4r)m^{31}}{31} + \frac{256(2r)m^{32}}{32}$

$S_mJ(D_mD_n) = \frac{9(5r)m^{18}}{2} + \frac{90(2r)m^{19}}{19} + 5(r)m^{20} + 6(2r)m^{25} + \frac{15(r)m^{30}}{2} + \frac{240(4r)m^{31}}{31} + 8(2r)m^{16}$

Putting m=1, we get $[S_mJ(D_mD_n)]|_{m=1}$

$= \frac{9(5r)}{2} + \frac{90(2r)}{19} + 5(r) + 6(2r) + \frac{15(r)}{2} + \frac{240(4r)}{31} + 8(2r)$

$NI(\mathcal{G}) = 103.441r.$

Sanskruti index:

$S(\mathcal{G}) = [S_m^3Q_{-2}J(D_m^3D_n^3)]|_{m=1}$

Take derivative of eq (5) to get D_m^3

$D_m^3 = 729(5r)m^9n^9 + 729(2r)m^9n^{10} + 1000(r)m^{10}n^{10} + 1000(2r)m^{10}n^{15} + 3375(r)m^{15}n^{15} + 3375(4r)m^{15}n^{16} + 4096(2r)m^{16}n^{16}$
…………… (13)

Now taking derivative of eq (6) to get D_n^3

$D_n^3 = 729(5r)m^9n^9 + 1000(2r)m^9n^{10} + 1000(r)m^{10}n^{10} + 3375(2r)m^{10}n^{15} + 3375(r)m^{15}n^{15} + 4096(4r)m^{15}n^{16} + 4096(2r)m^{16}n^{16}$
……………(14)

Multiplying eq (13) and (14), to get $D_m^3D_n^3$

$D_m^3D_n^3 = 531441(5r)m^9n^9 + 729000(2r)m^9n^{10} + 1000000(r)m^{10}n^{10} + 3375000(2r)m^{10}n^{15} + 11390625(r)m^{15}n^{15} + 13824000(4r)m^{15}n^{16} + 16777216(2r)m^{16}n^{16}$

Applying J on above eq to get,

$J(D_m^3D_n^3) = 531441(5r)m^{18} + 729000(2r)m^{19} + 1000000(r)m^{20} + 3375000(2r)m^{25} + 11390625(r)m^{30} + 13824000(4r)m^{31} + 16777216(2r)m^{32}$

Now applying $Q_{-2}J(D_m^3D_n^3)$ on above eq. As we know

$Q_\alpha f(x,y) = x^\alpha f(x,y)$

So,

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$$Q_{-2}J(D_m^3D_n^3) = 531441(5r)m^{16} + 729000(2r)m^{17} + 1000000(r)m^{18} + 3375000(2r)m^{23} + 11390625(r)m^{28} + 13824000(4r)m^{29} + 16777216(2r)m^{30}$$

Now, applying $S_m$on above equation to get,

$$S_mQ_{-2}J(D_m^3D_n^3) = \frac{531441(5r)m^{16}}{16} + \frac{729000(2r)m^{17}}{17} + \frac{1000000(r)m^{18}}{18} + \frac{3375000(2r)m^{23}}{23} + \frac{11390625(r)m^{28}}{28} + \frac{13824000(4r)m^{29}}{29} + \frac{16777216(2r)m^{30}}{30}$$

Again applying $S_m$on above equation to get,

$$S_m^2Q_{-2}J(D_m^3D_n^3) = \frac{531441(5r)m^{16}}{256} + \frac{729000(2r)m^{17}}{289} + \frac{1000000(r)m^{18}}{324} + \frac{3375000(2r)m^{23}}{529} + \frac{11390625(r)m^{28}}{784} + \frac{13824000(4r)m^{29}}{841} + \frac{16777216(2r)m^{30}}{900}$$

Again applying $S_m$on above eq to get this,

$$S_m^3Q_{-2}J(D_m^3D_n^3) = \frac{531441(5r)m^{16}}{4096} + \frac{729000(2r)m^{17}}{4913} + \frac{1000000(r)m^{18}}{5832} + \frac{3375000(2r)m^{23}}{12167} + \frac{11390625(r)m^{28}}{21952} + \frac{13824000(4r)m^{29}}{24389} + \frac{16777216(2r)m^{30}}{27000}$$

Putting m=1 to get $[S_m^3Q_{-2}J(D_m^3D_n^3)]|_{m=1}$

$$= \frac{531441(5r)}{4096} + \frac{729000(2r)}{4913} + \frac{1000000(r)}{5832} + \frac{3375000(2r)}{12167} + \frac{11390625(r)}{21952} + \frac{13824000(4r)}{24389} + \frac{16777216(2r)}{27000}$$

$S(\mathcal{G}) = 5872.106r.$

2.2 Convex Polytope S_r

We obtain the subdivision S (S_r) by adding additional vertex between each pair of neighboring vertices ofS_r. The S(S_r) consists of 13r vertices out which 8r vertices are of degree 2, 4r vertices are of degree 3 and r vertices are of degree 4. The line graph of subdivision of S_r is L(S (S_r)) for r = 8. The line graph of subdivision of S_r consists of 16r vertices out which 12r vertices are of degree 3 and 4r vertices are of degree 4.

Theorem 3. Let $\mathcal{G}$ be a line graph of subdivision of convex polytopeS_r. Then we have,

$$NM(\mathcal{G}; m, n) = (4r)m^9n^9 + (6r)m^9n^{10} + (r)m^9n^{11} + (r)m^{10}n^{10} + (2r)m^{10}n^{11} + (2r)m^{10}n^{15} + (2r)m^{11}n^{14} + (2r)m^{14}n^{14} + (4r)m^{15}n^{16} + (2r)m^{16}n^{16}$$

Proof:

The line graph of subdivision of convex polytope S_r (L(S(S_r))) has 26r number of edges. Its edge set can be partitioned as follows:

|{9,9} | = |{$uv \in E(\mathcal{G})$: δu = 9, δv = 9}| = 4r =ϒ(9,9) ,

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|{9,10} | = |{$uv \in E(\mathcal{G})$: δu = 9, δv = 10}| = 6r = ϒ (9,10) ,

|{9,11} | = |{$uv \in E(\mathcal{G})$: δu = 9, δv = 11}| = r = ϒ (9,11) ,

|{10,10} | = |{$uv \in E(\mathcal{G})$: δu = 10, δv = 10}| = r = ϒ (10,10) ,

|{10,11} | = |{$uv \in E(\mathcal{G})$: δu = 10, δv = 11}| =2r = ϒ (10,11) ,

|{10,15} | = |{$uv \in E(\mathcal{G})$: δu = 10, δv = 15}| = 2r = ϒ (10,15) ,

|{11,14} | = |{$uv \in E(\mathcal{G})$: δu = 11, δv = 14}| = 2r = ϒ (11,14),

|{14,14} | = |{$uv \in E(\mathcal{G})$: δu = 14, δv = 14}| = 2r = ϒ (14,14),

|{15,16} | = |{$uv \in E(\mathcal{G})$: δu = 15, δv = 16}| = 4r = ϒ (15,16),

|{16,16} | = |{$uv \in E(\mathcal{G})$: δu = 16, δv = 16}| = 2r = ϒ (16,16).

From the definition, the NM-polynomial of $\mathcal{G}$ is obtained as follows.

$$NM(\mathcal{G}) = \sum_{i \le j} \Upsilon_{(i,j)}\, m^i n^j$$

$$= \gamma_{(9,9)} m^9 n^9 + \gamma_{(9,10)} m^9 n^{10} + \gamma_{(9,11)} m^9 n^{11} + \gamma_{(10,10)} m^{10} n^{10} + \gamma_{(10,11)} m^{10} n^{11} + \gamma_{(10,15)} m^{10} n^{15} + \gamma_{(11,14)} m^{11} n^{14} + \gamma_{(14,14)} m^{14} n^{14} + \gamma_{(15,16)} m^{15} n^{16} + \gamma_{(16,16)} m^{16} n^{16}$$

.

$$NM(\mathcal{G}; m, n) = (4r) m^9 n^9 + (6r) m^9 n^{10} + (r) m^9 n^{11} + (r) m^{10} n^{10} + (2r) m^{10} n^{11} + (2r) m^{10} n^{15} + (2r) m^{11} n^{14} + (2r) m^{14} n^{14} + (4r) m^{15} n^{16} + (2r) m^{16} n^{16}$$

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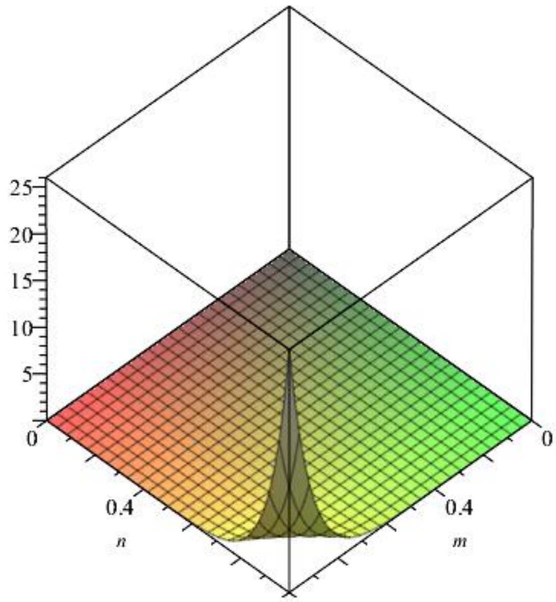

Figure 5: 3D plot of line graph of subdivision of convex polytope S_r

Theorem 4. Let $\mathcal{G}$ be a line graph of subdivision of convex polytope S_r. Then we have,

1. $M_1'(\mathcal{G}) = 612r,$
2. $M_2^*(\mathcal{G}) = 3755r,$
3. $F_N^*(\mathcal{G}) = 7594r,$
4. ${}^{nm}M_2(\mathcal{G}) = 0.2070r,$
5. $NR_\alpha(\mathcal{G}) = (3755)^\alpha r,$
6. $ND_3(\mathcal{G}) = 97012r,$
7. $ND_5(\mathcal{G}) = 52.592r,$
8. $NH(\mathcal{G}) = 2.3124r,$
9. $NI(\mathcal{G}) = 152.134r,$
10. $S(\mathcal{G}) = 7657.1632r.$

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Proof:

Let,

$NM(\mathcal{G};m,n) = (4r)m^9n^9 + (6r)m^9n^{10} + (r)m^9n^{11} + (r)m^{10}n^{10} + (2r)m^{10}n^{11} + (2r)m^{10}n^{15} + (2r)m^{11}n^{14} + (2r)m^{14}n^{14} + (4r)m^{15}n^{16} + (2r)m^{16}n^{16}$

……… (A)

Then we have,

$$M_1'(\mathcal{G}) = (D_m + D_n)|_{m=n=1}$$

$$D_m = m\left\{\frac{\partial\left(\mathrm{NM}(\mathcal{G};\mathrm{m},\mathrm{n})\right)}{\partial \mathrm{m}}\right\}$$

From equation (A), we get D_m

$D_m = 9(4r)m^9n^9 + 9(6r)m^9n^{10} + 9(r)m^9n^{11} + 10(r)m^{10}n^{10} + 10(2r)m^{10}n^{11} + 10(2r)m^{10}n^{15} + 11(2r)m^{11}n^{14} + 14(2r)m^{14}n^{14} + 15(4r)m^{15}n^{16} + 16(2r)m^{16}n^{16}$
…………… (1)

Similarly,

$$D_n = n\left\{\frac{\partial\left(\mathrm{NM}(\mathcal{G};\mathrm{m},\mathrm{n})\right)}{\partial \mathrm{n}}\right\}$$

$D_n = 9(4r)m^9n^9 + 10(6r)m^9n^{10} + 11(r)m^9n^{11} + 10(r)m^{10}n^{10} + 11(2r)m^{10}n^{11} + 15(2r)m^{10}n^{15} + 14(2r)m^{11}n^{14} + 14(2r)m^{14}n^{14} + 16(4r)m^{15}n^{16} + 16(2r)m^{16}n^{16}$
…………… (2)

Now adding eq (1) and (2) to get $(D_m + D_n)$

$(D_m + D_n) = 18(4r)m^9n^9 + 19(6r)m^9n^{10} + 20(r)m^9n^{11} + 20(r)m^{10}n^{10} + 21(2r)m^{10}n^{11} + 25(2r)m^{10}n^{15} + 25(2r)m^{11}n^{14} + 28(2r)m^{14}n^{14} + 31(4r)m^{15}n^{16} + 32\ (2r)m^{16}n^{16}$
…………… (3)

Putting m=n=1 in equation (3) to get this $(D_m + D_n)|_{m=n=1}$

$(D_m + D_n)|_{m=n=1} = 18(4r) + 19(6r) + 20(r) + 20(r) + 21(2r) + 25(2r) + 25(2r) + 28(2r) + 31(4r) + 32\ (2r)$

$= 72r + 114r + 20r + 20r + 42r + 50r + 50r + 56r + 124r + 64r$

$M_1'(\mathcal{G}) = 612r.$

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Neighborhood second Zagreb index:

$M_2^*(\mathcal{G}) = (D_m D_n)|_{m=n=1}$

By multiplying equation (1) and (2), we get

$D_m D_n = 81(4r)m^9n^9 + 90(6r)m^9n^{10} + 99(r)m^9n^{11} + 100(r)m^{10}n^{10} + 110(2r)m^{10}n^{11} + 150(2r)m^{10}n^{15} + 154(2r)m^{11}n^{14} + 196(2r)m^{14}n^{14} + 240(4r)m^{15}n^{16} + 256\,(2r)m^{16}n^{16}$

…………… (4)

Putting m=n=1 in eq (4) to get $(D_m D_n)|_{m=n=1}$

$(D_m D_n)|_{m=n=1} = 81(4r) + 90(6r) + 99(r) + 100(r) + 110(2r) + 150(2r) + 154(2r) + 196(2r) + 240(4r) + 256\,(2r)$

$= 324r + 540r + 99r + 100r + 220r + 300r + 308r + 392r + 960r + 512r$

$M_2^*(\mathcal{G}) = 3755r.$

Neighborhood forgotten topological index:

$F_N^*(\mathcal{G}) = (D_m^2 + D_n^2)|_{m=n=1}$

Now we take derivative of eq (1), to get D_m^2

$D_m^2 = 81(4r)m^9n^9 + 81(6r)m^9n^{10} + 81(r)m^9n^{11} + 100(r)m^{10}n^{10} + 100(2r)m^{10}n^{11} + 100(2r)m^{10}n^{15} + 121(2r)m^{11}n^{14} + 196(2r)m^{14}n^{14} + 225(4r)m^{15}n^{16} + 256\,(2r)m^{16}n^{16}$

…………… (5)

Now taking derivative of eq (2) to get D_n^2

$D_n^2 = 81(4r)m^9n^9 + 100(6r)m^9n^{10} + 121(r)m^9n^{11} + 100(r)m^{10}n^{10} + 121(2r)m^{10}n^{11} + 225(2r)m^{10}n^{15} + 196(2r)m^{11}n^{14} + 196(2r)m^{14}n^{14} + 256\,(4r)m^{15}n^{16} + 256\,(2r)m^{16}n^{16}$

…………… (6)

Adding equation (5) and (6), to get $(D_m^2 + D_n^2)$

$(D_m^2 + D_n^2) = 162(4r)m^9n^9 + 181(6r)m^9n^{10} + 202(r)m^9n^{11} + 200(r)m^{10}n^{10} + 221(2r)m^{10}n^{11} + 325(2r)m^{10}n^{15} + 317(2r)m^{11}n^{14} + 392(2r)m^{14}n^{14} + 481\,(4r)m^{15}n^{16} + 512\,(2r)m^{16}n^{16}$

…………… (7)

Now putting m=n=1 to get this $(D_m^2 + D_n^2)|_{m=n=1}$

$(D_m^2 + D_n^2)|_{m=n=1} = 162(4r) + 181(6r) + 202(r) + 200(r) + 221(2r) + 325(2r) + 317(2r) + 392(2r) + 481\,(4r) + 512\,(2r)$

$= 648r + 1086r + 202r + 200r + 442r + 650r + 634r + 784r + 1924r + 1024r$

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$F_N^*(\mathcal{G}) = 7594r.$

Neighborhood second modified Zagreb index:

$^{nm}M_2(\mathcal{G}) = (S_m S_n)|_{m=n=1}$

As we know,

$$S_m = \int_0^m \frac{f(t,n)}{t} dt$$

Applying this on equation (A), to get S_m

$$S_m = \int_0^m \{\frac{(4r)t^9n^9}{t} + \frac{(6r)t^9n^{10}}{t} + \frac{(r)t^9n^{11}}{t} + \frac{(r)t^{10}n^{10}}{t} + \frac{(2r)t^{10}n^{11}}{t} + \frac{(2r)t^{10}n^{15}}{t} + \frac{(2r)t^{11}n^{14}}{t} + \frac{(2r)t^{14}n^{14}}{t} + \frac{(4r)t^{15}n^{16}}{t} + \frac{(2r)t^{16}n^{16}}{t}\}dt$$

$$= \int_0^m \{(4r)t^8n^9 + (6r)t^8n^{10} + (r)t^8n^{11} + (r)t^9n^{10} + (2r)t^9n^{11} + (2r)t^9n^{15} + (2r)t^{10}n^{14} + (2r)t^{13}n^{14} + (4r)t^{14}n^{16} + (2r)t^{15}n^{16}\}$$
dt

$$= |\frac{(4r)t^9n^9}{9} + \frac{(6r)t^9n^{10}}{9} + \frac{(r)t^9n^{11}}{9} + \frac{(r)t^{10}n^{10}}{10} + \frac{(2r)t^{10}n^{11}}{10} + \frac{(2r)t^{10}n^{15}}{10} + \frac{(2r)t^{11}n^{14}}{11} + \frac{(2r)t^{14}n^{14}}{14} + \frac{(4r)t^{15}n^{16}}{15} + \frac{(2r)t^{16}n^{16}}{16}|_0^m$$

$$S_m = \frac{(4r)m^9n^9}{9} + \frac{(6r)m^9n^{10}}{9} + \frac{(r)m^9n^{11}}{9} + \frac{(r)m^{10}n^{10}}{10} + \frac{(2r)m^{10}n^{11}}{10} + \frac{(2r)m^{10}n^{15}}{10} + \frac{(2r)m^{11}n^{14}}{11} + \frac{(2r)m^{14}n^{14}}{14} + \frac{(4r)m^{15}n^{16}}{15} + \frac{(2r)m^{16}n^{16}}{16}$$
…………. (8)

Similarly following same procedure we can get S_n

$$S_n = \int_0^n \frac{f(m,)}{t} dt$$

$$S_n = \frac{(4r)m^9n^9}{9} + \frac{(6r)m^9n^{10}}{10} + \frac{(r)m^9n^{11}}{11} + \frac{(r)m^{10}n^{10}}{10} + \frac{(2r)m^{10}n^{11}}{11} + \frac{(2r)m^{10}n^{15}}{15} + \frac{(2r)m^{11}n^{14}}{14} + \frac{(2r)m^{14}n^{14}}{14} + \frac{(4r)m^{15}n^{16}}{16} + \frac{(2r)m^{16}n^{16}}{16}$$
…………...(9)

Multiplying equation (8) and (9) to get $(S_m S_n)$

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$$(S_mS_n) = \frac{(4r)m^9n^9}{81} + \frac{(6r)m^9n^{10}}{90} + \frac{(r)m^9n^{11}}{99} + \frac{(r)m^{10}n^{10}}{100} + \frac{(2r)m^{10}n^{11}}{110} + \frac{(2r)m^{10}n^{15}}{150} + \frac{(2r)m^{11}n^{14}}{154} + \frac{(2r)m^{14}n^{14}}{196} + \frac{(4r)m^{15}n^{16}}{240} + \frac{(2r)m^{16}n^{16}}{256}$$

Putting m=n=1 to get $(S_mS_n)|_{m=n=1}$

$$(S_mS_n)|_{m=n=1} = \frac{(4r)}{81} + \frac{(6r)}{90} + \frac{(r)}{99} + \frac{(r)}{100} + \frac{(2r)}{110} + \frac{(2r)}{150} + \frac{(2r)}{154} + \frac{(2r)}{196} + \frac{(4r)}{240} + \frac{(2r)}{256}$$

$$^{nm}M_2(\mathcal{G}) = 0.2070r.$$

Neighborhood general Randic index:

$$NR_\alpha(\mathcal{G}) = (D_m^\alpha D_n^\alpha)|_{m=n=1}$$

Using results from equation (4), we get

$$D_m^\alpha D_n^\alpha = (81)^\alpha(4r)m^9n^9 + (90)^\alpha(6r)m^9n^{10} + (99)^\alpha(r)m^9n^{11} + (100)^\alpha(r)m^{10}n^{10} + (110)^\alpha(2r)m^{10}n^{11} + (150)^\alpha(2r)m^{10}n^{15} + (154)^\alpha(2r)m^{11}n^{14} + (196)^\alpha(2r)m^{14}n^{14} + (240)^\alpha(4r)m^{15}n^{16} + (256)^\alpha\,(2r)m^{16}n^{16}$$

Putting m=n=1 in above equation we can get $(D_m^\alpha D_n^\alpha)|_{m=n=1}$

$$(D_m^\alpha D_n^\alpha)|_{m=n=1} = (81)^\alpha(4r) + (90)^\alpha(6r) + (99)^\alpha(r) + (100)^\alpha(r) + (110)^\alpha(2r) + (150)^\alpha(2r) + (154)^\alpha(2r) + (196)^\alpha(2r) + (240)^\alpha(4r) + (256)^\alpha\,(2r)$$

$$NR_\alpha(\mathcal{G}) = (3755)^\alpha r.$$

Third ND_e index:

$$ND_3(\mathcal{G}) = [D_mD_n(D_m + D_n)]|_{m=n=1}$$

By multiplying results of equation (3) and (4), we get that

$$D_mD_n(D_m + D_n) = 1458(4r)m^9n^9 + 1710(6r)m^9n^{10} + 1980(r)m^9n^{11} + 2000(r)m^{10}n^{10} + 2310(2r)m^{10}n^{11} + 3750(2r)m^{10}n^{15} + 3850(2r)m^{11}n^{14} + 5488(2r)m^{14}n^{14} + 7440(4r)m^{15}n^{16} + 8192\,(2r)m^{16}n^{16}$$

Putting m=n=1 to get $[D_mD_n(D_m + D_n)]|_{m=n=1}$

$$[D_mD_n(D_m + D_n)]|_{m=n=1} = 1458(4r) + 1710(6r) + 1980(r) + 2000(r) + 2310(2r) + 3750(2r) + 3850(2r) + 5488(2r) + 7440(4r) + 8192\,(2r)$$

$$= 5832r + 10260r + 1980r + 2000r + 4620r + 7500r + 7700r + 10976r + 29760r + 16384r$$

$$ND_3(\mathcal{G}) = 97012r.$$

.

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Fifth ND_e index:

$ND_5(\mathcal{G}) = (D_mS_n + S_mD_n)|_{m=n=1}$

Multiplying equation (1) and (9), we get D_mS_n

$$D_mS_n = (4r)m^9n^9 + \frac{9(6r)m^9n^{10}}{10} + \frac{9(r)m^9n^{11}}{11} + (r)m^{10}n^{10} + \frac{10(2r)m^{10}n^{11}}{11} + \frac{10(2r)m^{10}n^{15}}{15} + \frac{11(2r)m^{11}n^{14}}{14} + (2r)m^{14}n^{14} + \frac{15(4r)m^{15}n^{16}}{16} + (2r)m^{16}n^{16}$$

…………… (10)

Now multiplying equation (2) and (8) to get S_mD_n

$$S_mD_n = (4r)m^9n^9 + \frac{10(6r)m^9n^{10}}{9} + \frac{11(r)m^9n^{11}}{9} + (r)m^{10}n^{10} + \frac{11(2r)m^{10}n^{11}}{10} + \frac{15(2r)m^{10}n^{15}}{10} + \frac{14(2r)m^{11}n^{14}}{11} + (2r)m^{14}n^{14} + \frac{16(4r)m^{15}n^{16}}{15} + (2r)m^{16}n^{16}$$

…………… (11)

Now adding equation (10) and (11), to get $(D_mS_n + S_mD_n)$

$$(D_mS_n + S_mD_n) = 2(4r)m^9n^9 + \frac{181(6r)m^9n^{10}}{90} + \frac{202(r)m^9n^{11}}{99} + 2(r)m^{10}n^{10} + \frac{221(2r)m^{10}n^{11}}{110} + \frac{13(2r)m^{10}n^{15}}{6} + \frac{317(2r)m^{11}n^{14}}{154} + 2(2r)m^{14}n^{14} + \frac{481(4r)m^{15}n^{16}}{240} + 2(2r)m^{16}n^{16}$$

Putting m=n=1 to get, $(D_mS_n + S_mD_n)|_{m=n=1}$

$$(D_mS_n + S_mD_n)|_{m=n=1} = 2(4r) + \frac{181(6r)}{90} + \frac{202(r)}{99} + 2(r)m^{10}n^{10} + \frac{221(2r)}{110} + \frac{13(2r)}{6} + \frac{317(2r)}{154} + 2(2r) + \frac{481(4r)}{240} + 2(2r)$$

$ND_5(\mathcal{G}) = 52.592r$

Neighborhood Harmonic index:

$NH(\mathcal{G}) = 2S_mJ[NM(\mathcal{G})]|_{m=1}$

As we know, $Jf(m,n) = f(m,m)$

Now, we are going to apply J on eq (A).

$$J(NM(\mathcal{G})) = (4r)m^{18} + (6r)m^{19} + (r)m^{20} + (r)m^{20} + (2r)m^{21} + (2r)m^{25} + (2r)m^{25} + (2r)m^{28} + (4r)m^{31} + (2r)m^{32}$$

Now finding $S_mJ[NM(\mathcal{G})]$ of above eq

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$$S_mJ[NM(\mathcal{G})] = \frac{(4r)m^{18}}{18} + \frac{(6r)m^{19}}{19} + \frac{(r)m^{20}}{20} + \frac{(r)m^{20}}{20} + \frac{(2r)m^{21}}{21} + \frac{(2r)m^{25}}{25} + \frac{(2r)m^{25}}{25} + \frac{(2r)m^{28}}{28} + \frac{(4r)m^{31}}{31} + \frac{(2r)m^{32}}{32}$$

Now calculating $2S_mJ[NM(\mathcal{G})]$

$$2S_mJ[NM(\mathcal{G})] = \frac{(4r)m^{18}}{9} + \frac{2(6r)m^{19}}{19} + \frac{(r)m^{20}}{10} + \frac{(r)m^{20}}{10} + \frac{2(2r)m^{21}}{21} + \frac{2(2r)m^{25}}{25} + \frac{2(2r)m^{25}}{25} + \frac{(2r)m^{28}}{14} + \frac{2(4r)m^{31}}{31} + \frac{(2r)m^{32}}{16}$$

Putting m=1, we get $2S_mJ[NM(\mathcal{G})]|_{m=1}$

$$2S_mJ[NM(\mathcal{G})]|_{m=1} = \frac{(4r)}{9} + \frac{2(6r)}{19} + \frac{(r)}{10} + \frac{(r)}{10} + \frac{2(2r)}{21} + \frac{2(2r)}{25} + \frac{2(2r)}{25} + \frac{(2r)}{14} + \frac{2(4r)}{31} + \frac{(2r)}{16}$$

$$NH(\mathcal{G}) = 2.3124r$$

Neighborhood inverse sum index:

$$NI(\mathcal{G}) = [S_mJ(D_mD_n)]|_{m=1}$$

Applying J on eq (4)

$$J(D_mD_n) = 81(4r)m^{18} + 90(6r)m^{19} + 99(r)m^{20} + 100(r)m^{20} + 110(2r)m^{21} + 150(2r)m^{25} + 154(2r)m^{25} + 196(2r)m^{28} + 240(4r)m^{31} + 256(2r)m^{32}$$

Finding $S_mJ(D_mD_n)$ of above eq,

$$S_mJ(D_mD_n) = \frac{81(4r)m^{18}}{18} + \frac{90(6r)m^{19}}{19} + \frac{99(r)m^{20}}{20} + \frac{100(r)m^{20}}{20} + \frac{110(2r)m^{21}}{21} + \frac{150(2r)m^{25}}{25} + \frac{154(2r)m^{25}}{25} + \frac{196(2r)m^{28}}{28} + \frac{240(4r)m^{31}}{31} + \frac{256(2r)m^{32}}{32}$$

Putting m=,1 to get $[S_mJ(D_mD_n)]|_{m=1}$

$$[S_mJ(D_mD_n)]|_{m=1} = \frac{81(4r)}{18} + \frac{90(6r)}{19} + \frac{99(r)}{20} + \frac{100(r)}{20} + \frac{110(2r)}{21} + \frac{150(2r)}{25} + \frac{154(2r)}{25} + \frac{196(2r)}{28} + \frac{240(4r)}{31} + \frac{256(2r)}{32}$$

$$NI(\mathcal{G}) = 152.134r$$

Sanskruti index:

$$S(\mathcal{G}) = [S_m^3Q_{-2}J(D_m^3D_n^3)]|_{m=1}$$

Take derivative of equation (5), to get D_m^3

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$D_m^3 = 729(4r)m^9n^9 + 729(6r)m^9n^{10} + 729(r)m^9n^{11} + 1000(r)m^{10}n^{10} + 1000(2r)m^{10}n^{11} + 1000(2r)m^{10}n^{15} + 1331(2r)m^{11}n^{14} + 2744(2r)m^{14}n^{14} + 3375(4r)m^{15}n^{16} + 4096\,(2r)m^{16}n^{16}$

………….... (12)

Similarly, now taking derivative of equation (6), to get D_n^3

$D_n^3 = 729(4r)m^9n^9 + 1000(6r)m^9n^{10} + 1331(r)m^9n^{11} + 1000(r)m^{10}n^{10} + 1331(2r)m^{10}n^{11} + 3375(2r)m^{10}n^{15} + 2744(2r)m^{11}n^{14} + 2744(2r)m^{14}n^{14} + 4096(4r)m^{15}n^{16} + 4096\,(2r)m^{16}n^{16}$

………….... (13)

Multiplying equation (12) and (13), to get $D_m^3D_n^3$

$D_m^3D_n^3 = 531441(4r)m^9n^9 + 729000(6r)m^9n^{10} + 970299(r)m^9n^{11} + 1000000(r)m^{10}n^{10} + 1331000(2r)m^{10}n^{11} + 3375000(2r)m^{10}n^{15} + 3652264(2r)m^{11}n^{14} + 7529536(2r)m^{14}n^{14} + 13824000(4r)m^{15}n^{16} + 16777216\,(2r)m^{16}n^{16}$

Apply J on above eq,

$J(D_m^3D_n^3) = 531441(4r)m^{18} + 729000(6r)m^{19} + 970299(r)m^{20} + 1000000(r)m^{20} + 1331000(2r)m^{21} + 3375000(2r)m^{25} + 3652264(2r)m^{25} + 7529536(2r)m^{28} + 13824000(4r)m^{31} + 16777216\,(2r)m^{32}$

Now applying Q_{-2} on above eq, to get this

$Q_{-2}J(D_m^3D_n^3) = 531441(4r)m^{16} + 729000(6r)m^{17} + 970299(r)m^{18} + 1000000(r)m^{18} + 1331000(2r)m^{19} + 3375000(2r)m^{23} + 3652264(2r)m^{23} + 7529536(2r)m^{26} + 13824000(4r)m^{29} + 16777216\,(2r)m^{30}$

Now calculating $S_mQ_{-2}J(D_m^3D_n^3)$

$S_mQ_{-2}J(D_m^3D_n^3) = \frac{531441(4r)m^{16}}{16} + \frac{729000(6r)m^{17}}{17} + \frac{970299(r)m^{18}}{18} + \frac{1000000(r)m^{18}}{18} + \frac{1331000(2r)m^{19}}{19} + \frac{3375000(2r)m^{23}}{23} + \frac{3652264(2r)m^{23}}{23} + \frac{7529536(2r)m^{26}}{26} + \frac{13824000(4r)m^{29}}{29} + \frac{16777216\,(2r)m^{30}}{30}$

Now finding, $S_m^2Q_{-2}J(D_m^3D_n^3)$

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$$S_m^2 Q_{-2} J(D_m^3 D_n^3) = \frac{531441(4r)m^{16}}{256} + \frac{729000(6r)m^{17}}{289} + \frac{970299(r)m^{18}}{324} + \frac{1000000(r)m^{18}}{324} + \frac{1331000(2r)m^{19}}{361} + \frac{3375000(2r)m^{23}}{529} + \frac{3652264(2r)m^{23}}{529} + \frac{7529536(2r)m^{26}}{676} + \frac{13824000(4r)m^{29}}{841} + \frac{16777216\,(2r)m^{30}}{900}$$

Now we can calculate $S_m^3 Q_{-2} J(D_m^3 D_n^3)$

$$S_m^3 Q_{-2} J(D_m^3 D_n^3) = \frac{531441(4r)m^{16}}{4096} + \frac{729000(6r)m^{17}}{4913} + \frac{970299(r)m^{18}}{5832} + \frac{1000000(r)m^{18}}{5832} + \frac{1331000(2r)m^{19}}{6859} + \frac{3375000(2r)m^{23}}{12167} + \frac{3652264(2r)m^{23}}{12167} + \frac{7529536(2r)m^{26}}{17576} + \frac{13824000(4r)m^{29}}{24389} + \frac{16777216\,(2r)m^{30}}{27000}$$

Putting m=1, to get

$$[S_m^3 Q_{-2} J(D_m^3 D_n^3)]|_{m=1} = \frac{531441(4r)}{4096} + \frac{729000(6r)}{4913} + \frac{970299(r)}{5832} + \frac{1000000(r)}{5832} + \frac{1331000(2r)}{6859} + \frac{3375000(2r)}{12167} + \frac{3652264(2r)}{12167} + \frac{7529536(2r)}{17576} + \frac{13824000(4r)}{24389} + \frac{16777216\,(2r)}{27000}$$

$S(\mathcal{G}) = 7657.1632r$.

2.3. Graphical Representation

In this section, we will discuss results of convex polytopes, T_r areS_r, graphically. Where, red color indicated the reading of T_r while green color represented S_r. All indices react differently in each compound under consideration. Through graphical representations, we compare the behavior of the Neighborhood degree sum indices that correlate for both the T_r and S_r.

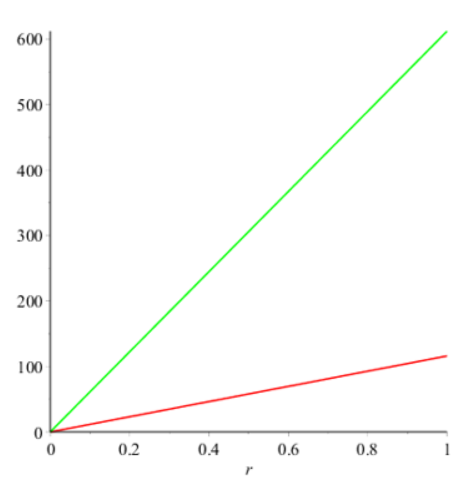

Figure 6: $M_1'(\mathcal{G})$ for T_r and S_r

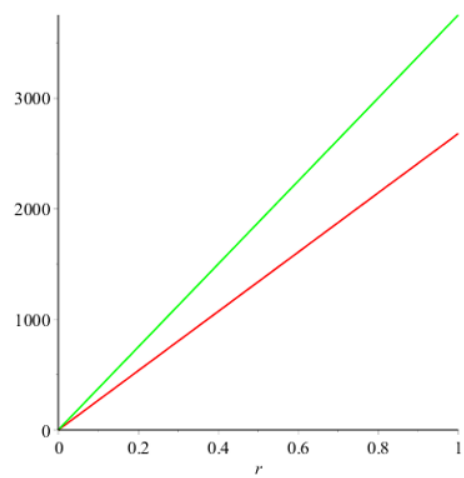

Figure 7: $M_2^*(\mathcal{G})$ for T_r and S_r

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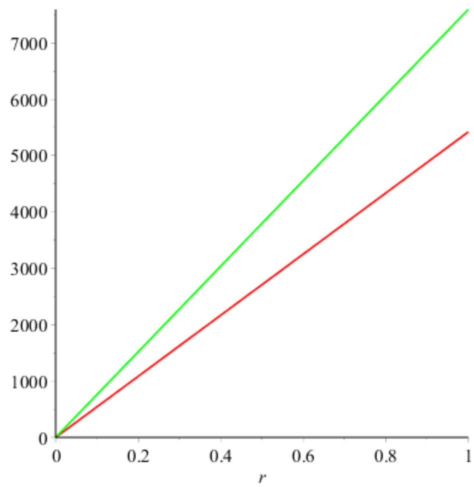

Figure 7: $F_N^*(\mathcal{G})$ **for** T_r **and** S_r

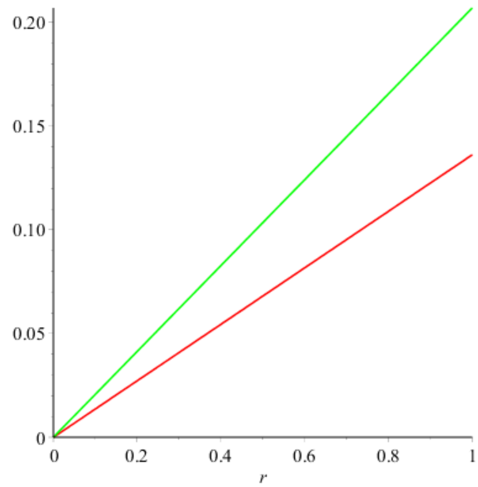

Figure 8: ${}^{nm}M_2(\mathcal{G})$ **for** T_r **and** S_r

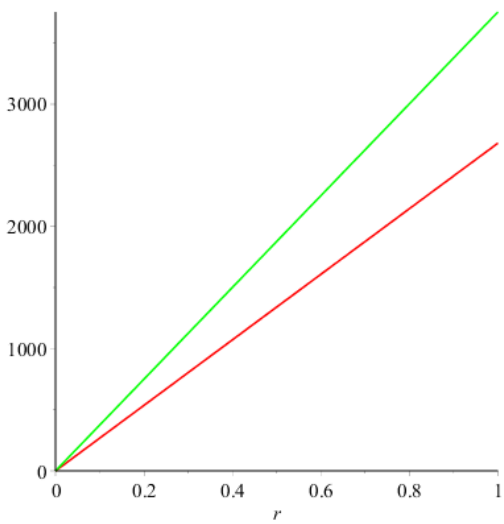

Figure 9. $NR_\alpha(\mathcal{G})$ **for** T_r **and** S_r

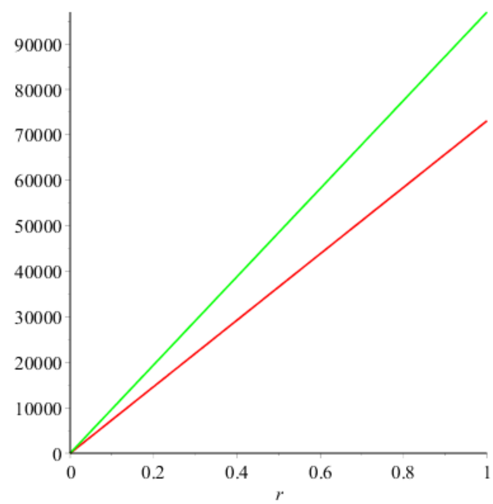

Figure 10. $ND_3(\mathcal{G})$ **for** T_r **and** S_r

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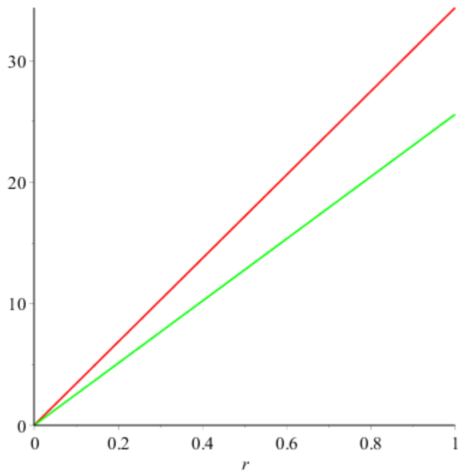

Figure 10. $ND_5(\mathcal{G})$ **for** T_r **and** S_r

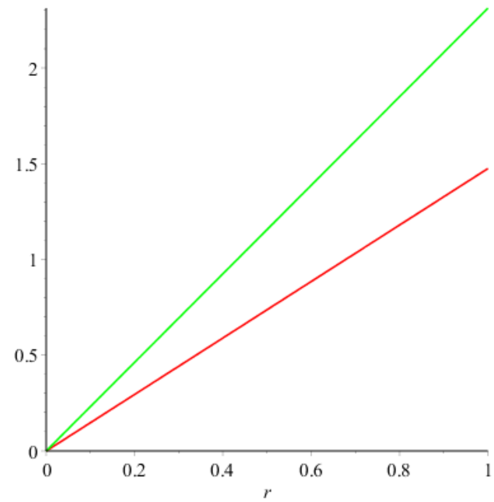

Figure 11: $NH(\mathcal{G})$ **for** T_r **and** S_r

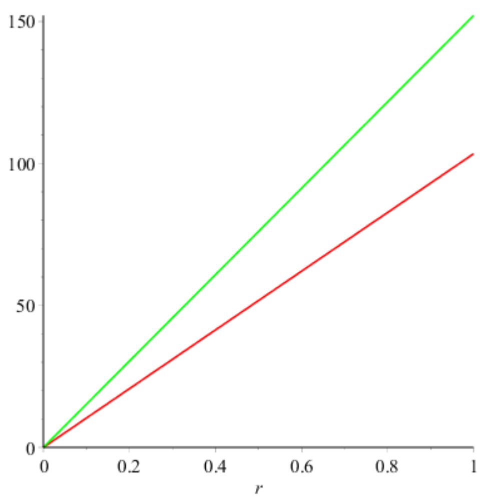

Figure 12. $NI(\mathcal{G})$ **for** T_r **and** S_r

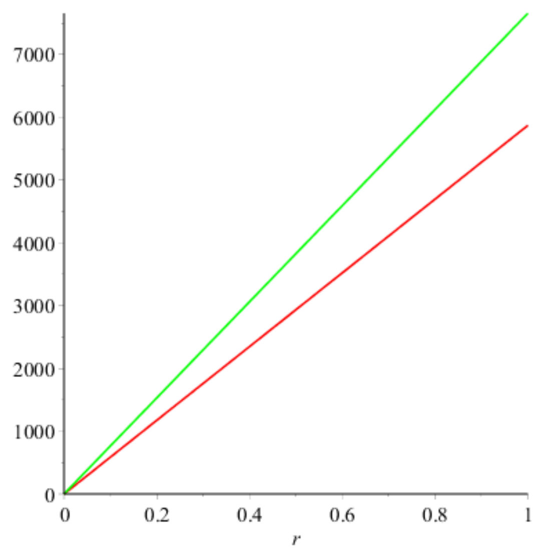

Figure 13. $S(\mathcal{G})$ **for** T_r **and** S_r

CONCLUSION

Convex polytopes are key mathematical items that have been researched since antiquity. Here, we calculate the newly introduced neighborhood M polynomials for line graph of subdivision of convex polytopes T_r and S_r. From the neighborhood M-polynomial, some neighborhood degree based topological indices are recovered. Our result will be helpful in non-exact quantitative structure / activity relationship. Furthermore, the importance of this research is to uncover fresh findings that will aid in the development of more exact and accurate estimates in the field of QSPR and QSAR.

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