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TOPOLOGICAL ANALYSIS EMPOWERED BRIDGE NETWORK VARIANTS BY DHARWAD INDICES

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Abstract

A bridge network is a network consisting of two or more computer networks bridged together. These are applicable in many fields like interconnections networks of memory, microprocessors, power generations, and chemical compounds Secondly, any number that can be uniquely determined by a graph is called a graph invariant. Bridge networks are directly related to graph invariants as these networks can be expressed in terms of a graph. During the last few decades, countless mathematical graph invariants have been characterized and utilized for correlation analysis. However, no reliable examination has been embraced to decide, how much these invariants are related to a network graph or computer networks. In this paper, the study will discuss three different variants of bridge networks like the tree, cyclic and fully connected networks embedded with bus networks and calculate them with the help of Dharwad Indices in the context of topologies.

Index Terms: Bridge networks, network graph, invariants, maple, Dharwad indices, interconnections networks, Computer Architecture

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1. INTRODUCTION

In 2021 V. R Kulli introduced some topological degree-based indices immediately after the Gutman's Sombor indices. These indices are called Dharwad Indices. It has a few other forms like reduced Dharwad , reduced Dharwad exponential, and δ-Dharwad index which are used to solve the topology of aromatic compounds [1]. These indices are used to solve the topology of a bridge graph very effectively and efficiently and find the lower bounds and upper bounds of three variants of a bridge network or graph. As another arising science, cheminformatics is identified with chemistry, mathematics, and computer sciences, whose significant segments incorporate Quantitative structure-activity (QSAR) and structure-property relationships (QSPR), and the segments can add to the examination of physicochemical properties of synthetic mixtures. As a numeric amount, a topological index is firmly identified with a graph that is invariant under graph automorphism and can portray the geography of a graph. Various uses of graph theory can be found in underlying chemistry [2]. Its first notable application in chemistry was the investigation of paraffin edges of boiling over by Wiener. Different topological records were presented following this examination that clarified physical-synthetic properties. The headway of large-scale integrated circuit innovation has empowered the development of complex interconnected networks. Graph theory gives a key apparatus to designing and evaluating such networks. Connected networks and graph theory gives a detailed comprehension of these connected themes. The architecture of an interconnected network is addressed by a graph, where nodes address the processors and edges address the connections between processors. Electric power organizations need to ceaselessly screen the condition of their frameworks on account of the voltage size at loads and the machine stage point at generators. In the electric power framework, a vertex addresses an electric hub and an edge addresses a transmission line joining two electrical nodes. Chemical graph theory is a part of numerical science wherein we apply devices of graph theory to demonstrate the chemical wonder numerically. In-network vertex is addressed network hub like PC, switch, switch some other gadget and an edge addresses a network path through which transmission completed. This theory contributes a noticeable job in the fields of chemical sciences [3].

This examination gives a basis to understand the profound topologies of some important networks and how these networks can be developed based on the best topological properties. This feature also gives potential assistance to scholars to contemplate network characteristics better. For additional work, if the related networks are replaced by different networks, we can also calculate and get the comparing formulas. A topological record is planned by transforming a network structure into a number. Originally, the main capacity of optical transpose interconnection system (OTIS) networks was to offer productive availability for new optoelectronic computer architectures that profit from both optical and electronic advancements with the

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assistance of topological records. An interconnection network's structure can be mathematically demonstrated by a graph. The geography of a graph decides how vertices are associated by edges. From the geography of a network, certain properties can easily be resolved. Maximum distance is resolved between any two hubs in the network. The quantity of connections associated with a hub decides the level of that hub. Topological Invariants (TIs) numerically portrayed the connectedness patterns (structure) between the hubs or actors in a network. So we can build a mind-boggling network of legal systems associating laws (hubs) that regulate normal biological subjects for instance. QSAR and QSPR provided the foundation for these models. A final remark is that the utilization of the measurement in the network plane facilitates a quantitative evaluation of various geography safeguarding mapping algorithms [4]. In this article, we will compute different indices of bridge networks. The results will play a vital role in determining the properties of these networks and their uses in the computer industry, electronics, chemistry, pharmacy, etc. We will also perform index analysis on certain networks which may be beneficial for people who are working in the field of computer science, mathematics and chemistry also. In the end, we formulate a mathematical formula to construct a computer network or processor design or chemical compound, check the properties of the concerning before and also check the feasibility of anyone which discuss earlier [5]. In this paper first, we discuss objectives, significance, and research gap, secondly review the literature, thirdly discuss research gap and research questions, in the fourth section we discuss research methodology and limitations, in the fifth section we analyze data, and in the last section we write results and conclude the research.

2. LITERATURE REVIEW

A topological index or molecular descriptor is a numerical plan that can be applied to any graph which models some molecular structure. In light of the molecular descriptor, breaking down numerical qualities and advancing the examination of a few physicochemical properties of a molecule is helpful. Therefore, it is a viable strategy in supplanting arduous, costly, and tedious lab detailed trials. Molecular descriptors play a significant part in numerical chemistry; particularly during the examination of Quantitative structure-property relationships (QSPRs) and Quantitative structure-action relationships (QSARs) that give understanding into vivify impacts because of substance structures. In this paper, we have figured out some novel neighborhood rendition molecular descriptors for the two carbon Nano sheets and inferred equations for them. Our figured outcomes are most likely accommodating in understanding the geography of student Nano sheets. These registered indices have the best correlation with acentric variables and entropy, hence, are successful in QSPRs and QSARs investigation with strong precision[6].

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Topological indices (TI) (descriptors) of a molecular graph are especially valuable to study different physiochemical properties. It is additionally used to foster the quantitative structure-action relationship (QSAR), and quantitative structure-property relationship (QSPR) of the comparing synthetic compound. Different methods have been created to ascertain the TI of a graph. As of late a method of computing degree-based TI from M-polynomial has been presented. We have assessed different topological descriptors for 3-layered TiO_2gems utilizing M-polynomial. These descriptors are built with the end goal that it contains 3 variables (m, n, and t) each compared to a specific course. These 3 variables work with us to profoundly get the development of TiO_2 in 1 aspect (1D), 2 aspects (2D), and 3 aspects (3D) separately[7].

The target of network module location is to distinguish gatherings of hubs inside a network structure that are tightly connected. Hubs in a network regularly have attributes (also known as metadata) related to them. It is regularly attractive to distinguish gatherings of hubs that are tightly connected in the network structure, yet in addition have solid closeness in their attributes. Using property data in module discovery is a significant test since it requires connecting the structural network with property information. A Weighted Fast Greedy (WFG) algorithm for quality-based module recognition is proposed. WFG uses calculated relapse to bridge the structural and characteristic spaces. The calculated capacity normally underscores the relationship among attributes and network structure in a like manner and can be effectively deciphered. A bosom malignant growth application is introduced that interfaces a protein-protein cooperation network quality articulation information and an endurance result. This application exhibits the significance of inserting characteristic data into the local area recognition system on a bosom malignant growth dataset. Five modules were critical for endurance and they contained known pathways and markers for malignant growth, including cell cycle, p53 pathway, BRCA1, BRCA2, and AURKB, among others. While, neither the quality articulation information nor the network structure alone led to these malignant growth biomarkers and marks[8].

In the view of researchers, the headway of large-scale integrated circuit innovation has empathy for the development of complex interconnected networks. Graph theory gives a key apparatus to designing and evaluating such networks. Connected networks and Graph theory gives a detailed comprehension of these connected themes. The architecture of an interconnected network is addressed by a graph, where nodes address the processors and edges address the connections between processors. Electric force organizations need to ceaselessly screen the condition of their frameworks on account of the voltage size at loads and the machine stage point at generators. In the electric force framework, a vertex addresses an electric hub and an edge addresses a transmission line joining two electrical nodes. Chemical graph theory is a part of numerical science wherein they apply devices of graph theory to demonstrate the chemical wonder numerically. In-network vertex is addressed network

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hub like PC, switch, switch some other gadget and an edge addresses a network path through which transmission completed[9].

In this article, the study presented advancements bridging radio and optical transmissions for 5G networks. The RoF strategy can give consistent associations among radio and optical media, permitting adaptable utilization of the MFH and connect assurance in the MBH. FSO connections would likewise be an answer for reinforcement remote connections. RoF can be joined with frequency division multiplexing to give high-velocity direct cell correspondence for moving trains, and the SoF idea would give high-goal FOD radar discovery for well-being in air terminals. At last, we have talked about and shown simple RoF joins for hand-off innovation to mmW radio no man's lands, for example, in-vehicle and indoor applications, utilizing a 60-GHz SSB-RoF interface framework. A wide assortment of uses utilizing photonic gadgets that bridge wired and remote media has been illustrated. In any case, the related equipment has numerous varieties relying upon the last use. Coordinated equipment that can be deftly utilized in any of these situations is, therefore, exceptionally alluring. This sort of coordinated equipment would open numerous entryways for the incorporation of optics in remote access administrations like 5G and above[10][11].

This review presented the second Banhatti-Sombor index, second, decreased Banhatti-Sombor index, and second δ-Banhatti-Sombor index of a graph and has registered accurate equations for easy chair polyhex nanotubes, crisscross polyhex nanotubes and carbon Nano cone networks[12][13]. In this paper, researchers calculate the irregularity indices of honeycomb networks, hexagonal networks, oxide networks, and silicate networks. Researchers compared the results also. The results are very helpful in understanding the behavior of different computer networks and chemical networks. After understanding these formulas different researchers can construct their own best networks in chemistry and computer also[14].

In this paper, they stretch out this examination to interconnection networks and determine scientific shut aftereffects of General Randi c' index R (G) for various upsides of " α" for octagonal network, toroidal polyhex, and summed up a crystal. Interconnection networks of multiprocessors are regularly needed to interface a great many homogeneously reproduced processor-memory combines, every one of which is known as a handling hub. Rather than utilizing a common memory, all synchronization and correspondence between handling hubs for program execution are regularly done through message passing. Plan and utilization of multiprocessor interconnection networks have as of late attracted extensive consideration because of the accessibility of cheap, incredible microchips and memory chips. The octagonal, toroidal polyhex and summed-up crystal networks have been perceived as flexible interconnection networks for greatly equal registering. This is basically because of the way that these groups of networks have geographies that mirror the correspondence example of a wide assortment of characteristic issues. Toroidal polyhex networks have as of late gotten a

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ton of consideration for their better versatility to bigger networks, instead of more unpredictable networks, for example, hypercube [15].

This paper is intended to profit the understudies of computer science to acquire profundity information on graph theory and its importance with other subjects like operating systems, Networks, Databases, software engineering and so forth this paper zeroed in on the different applications of significant graph theory that have pertinence to the field of computer science and applications [16].

In turn, these fields have invigorated the improvement of numerous new diagram theoretical ideas and prompted many challenging graph theory issues. They can expect that the continued interplay between graph theory and numerous spaces of utilization will prompt significant new turns of events. The significant job of graph theory in computer applications is the improvement of graph calculations. [17].

3. RESEARCH METHODOLOGY

3.1 Objectives

The main objective is to investigate the topological invariants of bridge computer networks. It is also finding out the intensity of seriousness of topological indices in certain computer networks, like interconnection networks of processors, and in power interconnection networks, etc. It explains the Dharwad indices, their reduced form, and their benefits.

The study also develops mathematical formulas, so that we can check the topology, and performance of computer networks, processors interconnection, or power interconnection networks without doing/performing experiments before manufacturing.

3.2 Significance

The study initiates about Topological invariants, bridge networks and their uses in certain computer networks. It is also developing new and substantial results or formulas for developing above mentioned products because no satisfactory solution has been found till now due to the incremental and fast nature of computer networks and interconnection networks.

3.3 Method

The methodology deals with bridge network variants, their uses and empowerment in interconnection networks used in different products and networks. To find the answer to the research questions and sub-questions which have been discussed in the literature review. The method to develop topological invariants for computer certain networks. It compares different results used for modeling computer certain networks and proposed the best model after analysis and made available to the research community. Simulation tools are used for the verification and validation of results.

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In this systematic study, take an existing network, associate it with a graph and solve the topology of the graph with the help of topological invariants like the Dharwad index and its reduced form. The concerning results in the form of formulas will compare with existing results. These deduced results will apply to many other networks in the fields of computer networks, processor interconnection networks, memory interconnection networks, power interconnection networks, and image processing afterward.

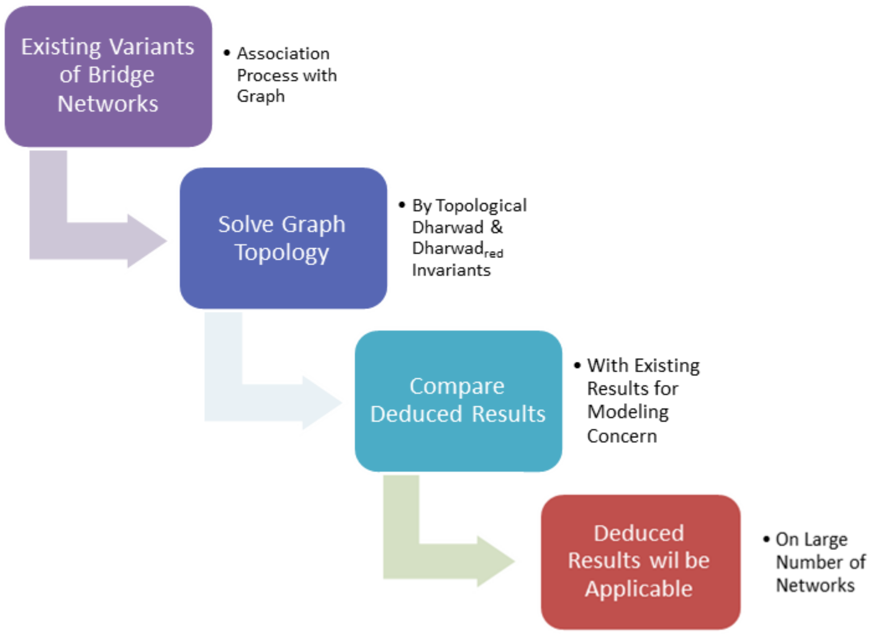

Fig. 1: Systematic methodology

Fig. 1 shows the flow of the systematic study of this article which takes existing bridge networks associates them with graph theory, solves the topology of the graph by Dharwad indices and compares the results, and deduced results will be used for modeling certain networks.

4. EXPERIMENTATION

In 2009, authors introduce the idea of bridge networks which is a combination of networks bridged together [18]. A bridge graph is a graph obtained from the number of graphs G1, G2, G3,...Gm by associate the vertices vi and vi + 1 by an edge∀, i = 1,2,..., m − 1 [19]. V.R Kulli introduced Dharwad Index and its reduced form which are given as [21]. These are denoted by D (G) and RD (G).

$$D(G) = \sum_{ue} \sqrt{du^3 + dv^3} \tag{1}$$

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The graph theory introduced many graph invariants and studied also but Eq. (1) shows the Dharwad index which will be used for the solution of the bridge network mentioned in Fig. 2. This invariant can calculate and find sharp upper bounds at the boundaries [22].

$$RD\ (G) = \sum_{ue} \sqrt{(du-1)^3 + (dv-1)^3} \qquad (2)$$

The graph theory introduced many graph invariants and studied also but Eq. (2) shows the Dharwad reduced index which will be used for the solution of the bridge network mentioned in Fig. 2. This invariant can calculate and find sharp lower bounds at the boundaries [23]. There are four edges of the bridge of trees ε1, ε2, ε3 and ε4. The edge partition is given in Fig. 2. of Gr (Ps, v) over Ps are ε1(1, 2), ε2(2, 2), ε3(2, 3) and ε4(3, 3) with frequencies R, 3R+2, R and R-3 respectively. The study found vertices of G are may be 1, 2, or 3 shown in Fig. 2. After the calculation, the number of edges formed is r, 3r+2 and r-3 [24].

4.1 Main Results

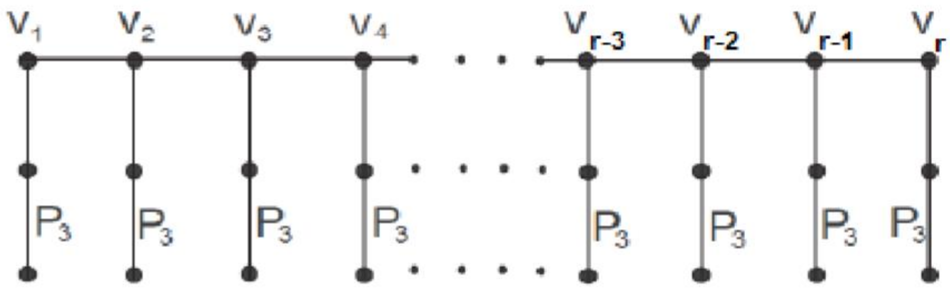

Fig. 2: Gr (Ps,v) over Ps

Bridge graph Gr (Ps, v) over Path. As we see from Fig. 2 vertex set name is V which is divided into four groups of subsets V1, V2, V3 andV4, Such that V = V1 +V2 +V3+V4. If the edge set is represented by E then, there are four distinct classes of edges in the network graph of bridge graph Gr (Ps, v) over the path of hybrid networks [25].

Theorem 4.1.1 Let *G* be a graph of $G_r\ (P_s,\ v)$ over P_s, then, *Dharwad* and *Dharwad* $_{red}$ indices are

$$D\ (G) = 15r + 8 + \sqrt{35}\, r + 3\sqrt{6}\,(r-3). \qquad (3)$$

$$RD_{red}\,(G) = 8r + \sqrt{2}\,(3r+2) - 12 \qquad (4)$$

Eq. 5 and Eq. 6 represent the proven results of the graph of the bridge network mentioned in Fig. 2.

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4.2 Investigation of Bridge Graphs by Dharwad Indices

Proof.

$D(G) = \sum_{ue} \sqrt{du^3 + dv^3}$

D (G) = $\sqrt{1^3 + 2^3}\,(r) + 9\sqrt{2^3 + 2^3}\,(3r + 2) + \sqrt{2^3 + 3^3}\,(r) - 5\sqrt{3^3 + 3^3}\,(r - 3)$

D (G) = $15r + 8 + \sqrt{35}\,r + 3\sqrt{6}\,(r - 3).$

$RD\ (G) = \sum_{ue} \sqrt{(du - 1)^3 + (dv - 1)^3}$

RD (G) = $\sqrt{(1 - 1)^3 + (1 - 1)^3}\,r + 5\sqrt{(2 - 1)^3 + (2 - 1)^3}\,(3r + 2)$

$+4\sqrt{(2 - 1)^3 + (3 - 1)^3}\,r + \sqrt{(3 - 1)^3 + (3 - 1)^3}(r - 3)$

RD (G) = $8r + \sqrt{2}\,(3r + 2) - 12$

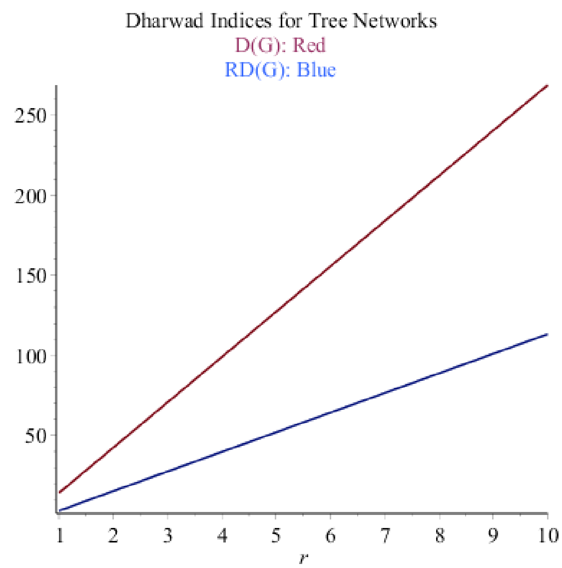

Fig. 3: Dharwad and *Dharwad* $_{red}$ for G_r (K_s, v) over K_s

Fig. 3 results show sharp lower and upper bounds Eq. (3) & (4) of Dharwad indices and their reduced form in red and blue lines respectively.

There are four edges of the bridge of cycles G_r ($C_{s,\ v}$) over C_s ε1, ε2, ε3 and ε4. The edge partition given in Fig. 4. Of G_r ($C_{s,\ v}$) over C_s is ε1 (2, 2), ε2 (2, 3), ε3 (2, 4) and ε4 (3, 4) with frequencies rs-2r, 4, 2r-4 and 2 respectively given in Fig. 4. The study found vertices of G are may be 2, 3 or 4 shown in Fig. 4. After the calculation, the number of edges formed is shown above.

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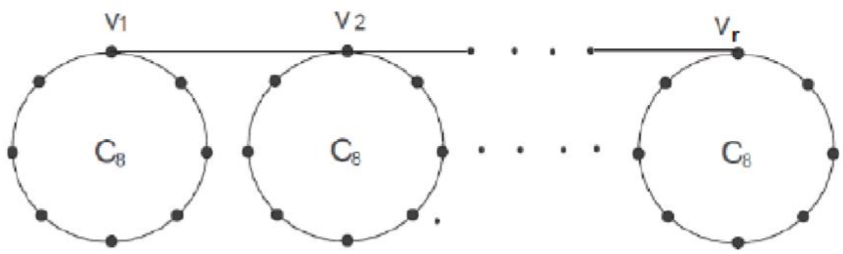

Fig. 4: G_r (C_s, v) over C_s [26]

4.3 Bridge graph Gr (Cs, v) over Cycle.

If V is the vertex set then by the observation of Figure 4, we can classify this vertex set into four subsets V1, V2, V3, and V4, Such that V = V1 +V2 +V3 +V4. If E (D2 (m)) represents the edge set. Bridge graph Gr (Cs, v) over the Cycle of hybrid networks has five distinct edges in the network graph as shown in Fig. 4.

4.4 Theorem 2.2

Let the study draw a G graph of G_r (C_s, v) over C_s, then *Dharwad* and *Dharwad* $_{red}$ *Indices* are

$$D(G) = 4rs - 8r - 4\sqrt{35} + 6\sqrt{2}(2r - 4) + 2\sqrt{91} + 8\sqrt{2}(r-3) \qquad (5)$$

$$RDred(G) = \sqrt{2}(rs - 2r) + 12 + 2\sqrt{7}(2r-4) + 2\sqrt{35} + 3\sqrt{6}(r-3) \qquad (6)$$

Eq. 5 and Eq. 6 represent the proven results of the graph of the cyclic bridge network mentioned in Fig. 4. With the help of the Dharwad Index and its reduced form.

4.5 Investigation of Bridge Graphs by Dharwad Indices

Proof.

$$D(G) = \sum_{ue} \sqrt{du^3 + dv^3}$$

$$D(G) = \sqrt{2^3+2^3}(rs-2r) + \sqrt{2^3+3^3}\,4 + \sqrt{2^3+4^3}(2r-4) - \sqrt{3^3+4^3}(2) + \sqrt{4^3+4^3}(r-3)$$

$$D(G) = 4rs - 8r - 4\sqrt{35} + 6\sqrt{2}(2r - 4) + 2\sqrt{91} + 8\sqrt{2}(r-3)$$

$$RD(G) = \sum_{ue} \sqrt{(du-1)^3 + (dv-1)^3}$$

$$RD(G) = \sqrt{(2-1)^3+(2-1)^3}(rs-2r) + \sqrt{(2-1)^3+(3-1)^3}(4)$$

$$+\sqrt{(2-1)^3+(4-1)^3}(2r-4) + \sqrt{(3-1)^3+(4-1)^3}(2)$$

$$+\sqrt{(4-1)^3+(4-1)^3}(r-3)$$

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$$RD\ (G)\ = \sqrt{2}\,(rs - 2r) + 12 + 2\sqrt{7}\,(2r - 4) + 2\sqrt{35} + 3\sqrt{6}\,(r - 3)$$

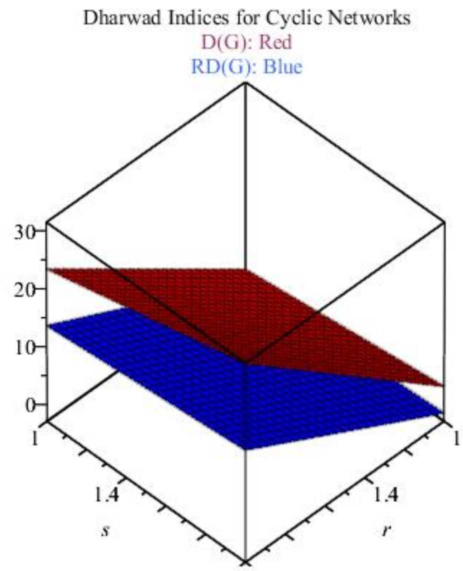

Fig. 5. Results with Dharwad and Dharwad reduced indices for G_r (K_s, v) over Ks

Fig. 5 shows the results (Equations 5 & 6) of Dharwad indices and their reduced form in red and blue color in the 3D version showing lower bounds and upper bounds of the bridge graph respectively.

There are five edges of the bridge of cycles G_r (K_s, v) over Ks ε1, ε2, ε3, ε4 and ε5. The edge partition given in Fig. 6. of G_r ($C_{s,\,v}$) over C_s are ε1(4, 5), ε2(4, s-1), ε3(5, 5) , ε4(5, s-1) and ε5(s-1, s-1) with frequencies 2, 2, r-2, r-2 and [rs(r-1)-2(r+1)]/2 respectively given in Fig. 6.

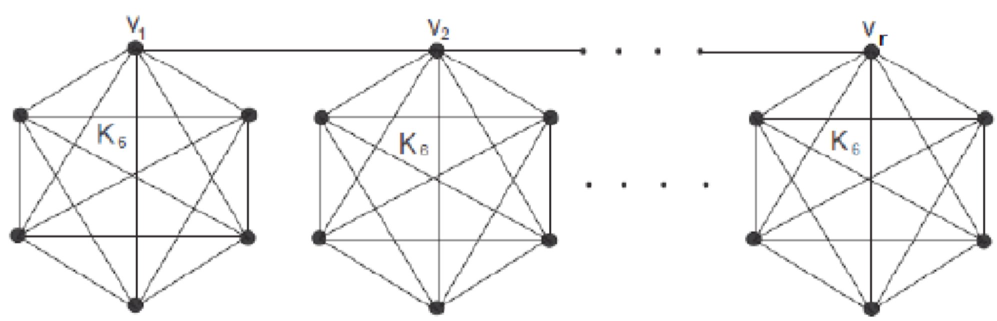

Fig. 6: G_r (K_s, v) over K_s

Fig. 6 shows the bridge networks in which bus networks and fully connected networks bridged together.

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4.6 Bridge graph Gr (Ks, v) over Complete Graph

According to Fig. 6, the vertex set of the network is V which can be divided into three distinct sets V1, V2, andV3, such that V = V1 +V2 +V3. If we consider the edge set named E which shows five distinct types of edges in a given network of fully connected networks Gr (Ks, v) over the complete graph of hybrid networks. Table 3, gives a detailed explanation of the edge set.

4.7 Theorem 2.3

Let G be a graph of **G_r (K_s, v)** over Ks. Then *Dharwad* and *Dharwad $_{red}$* indices are

$$D(G) = 6\sqrt{21}\, 2\sqrt{64+(s-1)^3} + 5\sqrt{10}(r-2) + \sqrt{125+(s-1)^3}(r-2) + \sqrt{2}\sqrt{(s-1)^3}(\tfrac{1}{2}rs(r-1)-r-1) \quad (7)$$

$$D(G) = 2\sqrt{91}\, 2\sqrt{27+(s-2)^3} + 8\sqrt{2}(r-2) + \sqrt{64+(s-2)^3}(r-2) + \sqrt{2}\sqrt{(s-2)^3}(\tfrac{1}{2}rs(r-1)-r-1) \quad (8)$$

Eq. 7 and Eq. 8 represent the proven results of the graph of Gr (Ks, v) over the complete graph mentioned in Fig. 6.

4.8 Investigation of Bridge Graphs by Dharwad Indices

Proof.

$$D(G) = \sum_{ue} \sqrt{du^3+dv^3}$$

$$D(G) = \sqrt{4^3+5^3}\,(2) + \sqrt{4^3+(s-1)^3}\,(2) + \sqrt{5^3+5^3}\,(r-2) - \sqrt{5^3+4^3}\,(2)$$

$$+\sqrt{(s-1)^3+(s-1)^3}\,(r-2) + \sqrt{5^3+(s-1)^3}\,(\mathrm{rs}(\mathrm{r}-1)-2(\mathrm{r}+1))/2$$

$$D(G) = 6\sqrt{21}\, 2\sqrt{64+(s-1)^3} + 5\sqrt{10}(r-2) + \sqrt{125+(s-1)^3}(r-2) + \sqrt{2}\sqrt{(s-1)^3}(\tfrac{1}{2}rs(r-1)-r-1)$$

$$RD(G) = \sum_{ue} \sqrt{(du-1)^3+(dv-1)^3}$$

$$RD(G) = \sqrt{(4-1)^3+(5-1)^3}\,(2) + \sqrt{(4-1)^3+(s-2)^3}\,(2)$$

$$+\sqrt{(5-1)^3+(5-2)^3}\,(r-2) + \sqrt{(5-1)^3+(s-2)^3}(r-2)$$

$$+\sqrt{(s-2)^3+(s-2)^3}\,(\mathrm{rs}(\mathrm{r}-1)-2(\mathrm{r}+1))/2$$

$$RD(G) = 2\sqrt{91}\, 2\sqrt{27+(s-2)^3} + 8\sqrt{2}(r-2) + \sqrt{64+(s-2)^3}(r-2) + \sqrt{2}\sqrt{(s-2)^3}(\tfrac{1}{2}rs(r-1)-r-1)$$

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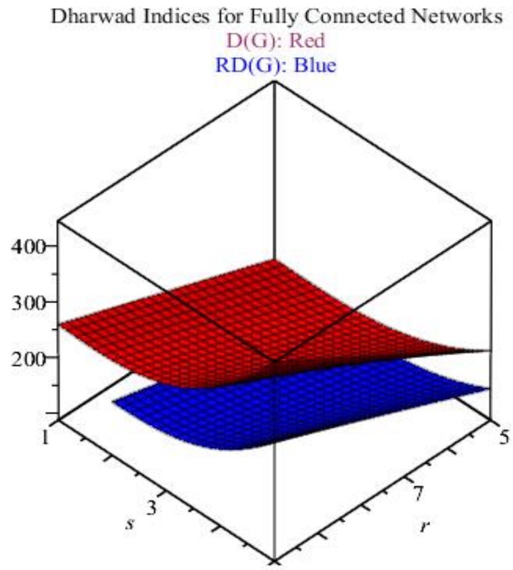

Figure 7: Results for Dharwad and Dharwad reduced indices for G_r (K_s, v) over Ks

Fig. 7 shows the results Eq. (7) & (8) of Dharwad indices and their reduced form in red and blue color in the 3D version showing lower bounds and upper bounds of the fully connected bridge graph respectively.

5. CONCLUSION

TIs tracked down various applications in numerous areas of computer science, math, informatics, software engineering, chemistry, biology, etc., but their supreme significant consumption is in the non-definite QSPR and QSAR. TIs are related to the arrangement of bridge networks utilized in the spine of the internet, memory interconnection networks, computer microchips, and compound structures. In this paper, the analysis examines the recently presented Dharwad invariants for three unique variations of bridge graphs or networks, for example, Gr (Ps, v), Gr (Cs, v), and Gr (Ks, v). Figures 3, 5, and 7 give the graphical portrayal of Dharwad indices and their reduced forms for tree, cyclic and fully connected bridge graphs of bridge networks. The deduced mathematical results can be used in the construction of these interconnection networks in different fields like computer science, chemistry, and electronics.

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CONFLICTS OF INTEREST

All authors showing no conflict regarding results, theme, discussion and conclusion.

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