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RESEARCH OF THE APPROXIMATE THEORY OF VIBRATIONAL PROCESSES OF DIFFERENT TYPES OF FLAT ELEMENTS

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Annotation

This paper presents the results of the study of natural and forced vibrations of flat elements, taking into account the layering of the element material, rheological viscous properties, environmental influences, deformable base, anisotropy, etc. Various formulations of boundary value problems of vibrations of a rectangular flat element are considered, both taking into account viscosity. and taking into account the above factors of a geometric and mechanical nature, which are trance dental frequency equations, which is reduced to algebraic ones, and the influence of both boundary conditions along the edges of a rectangular plate and the parameters of a geometric and mechanical nature on the frequencies of natural oscillations of rectangular flat elements is considered and the previous results are generalized for a rectangular plate, the material of which satisfies the viscoelastic Maxwell model. [One] When studying oscillatory processes in a solid deformable body, it is advisable to take the kernel of viscoelastic operators as regular, since only such operators describe instantaneous elasticity and then viscous flow, which is typical for deformable solids. Integro-differential equations with regular kernels are known to be equivalent to partial differential equations. Depending on the considered particular types of a flat element in the general solutions of a three-dimensional problem, the main unknown functions are chosen: displacements or deformations at points of a fixed plane of a flat element, in particular, in the middle plane of a plate of constant thickness. Displacements and stresses at an arbitrary point of a flat element are expressed in terms of the main unknown functions, which are determined from the boundary conditions on the surfaces of a flat element. The equations obtained for the main unknown functions and are the general equations for the vibration of a plane element, containing the derivatives of functions with respect to coordinates and time of any arbitrarily large order. General solutions are presented as power series over the thickness of a flat element. The general solution refers to an equation of the hyperbolic type, which describes the oscillatory and wave processes in a flat element. Restricting ourselves in the series of the general equation to a finite number of first terms, we obtain approximate equations for the vibration of one or another flat element. The proposed approach makes it possible to rigorously construct approximate theories of the oscillations of flat elements of various types.

Keywords: boundary conditions, deformation, flat element, movement, natural vibrations vibration theories*.*

1. INTRODUCTION

When solving applied problems of vibration of rectangular flat elements, a wide class of vibration problems arises associated with various boundary value problems: approximate vibration equations, various boundary conditions at the edges of a flat element, and initial conditions. In the theory of oscillations, an important point is the determination of the frequencies of natural oscillations, the solution of problems of forced oscillations of a flat element and the study of the propagation of harmonic waves in them.

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In problems of determining the frequencies of natural vibrations of flat elements hinged at the edges and on the basis of approximate theories obtained on the basis of hypotheses and assumptions of a mechanical and geometric nature, in particular, on the basis of approximate equations such as the parabolic Kirchhoff equation, which poorly describe the wave and oscillatory nature of behavior flat element under non-stationary external influences.

In the study of harmonic waves in deformable bodies, the concept of phase velocity is introduced as the rate of change in the state of the medium, while the phase velocity is expressed in terms of the frequencies of natural oscillations and therefore the study of the propagation of harmonic waves is directly related to the problems of determining the natural forms and frequencies of oscillations of flat elements limited in plan.

This paper presents the results of the study of natural and forced vibrations of flat elements, taking into account the layering of the element material, rheological viscous properties, environmental influences, deformable base, anisotropy, etc.

The influence of these factors greatly complicates the study of problems of natural and forced vibrations of a plane element, of the propagation of harmonic waves in them.

The work is devoted to the formulation of various boundary value problems of oscillations of a rectangular flat element, both taking into account viscosity. And taking into account the above factors of a geometric and mechanical nature.

2. FORMULATION OF THE PROBLEM

We confine ourselves to solving the task on the basis of an approximate equation of transverse oscillations of the fourth order [2]

$$P_0(W)+p\frac{\partial^2 W}{\partial t^2}+\frac{h^2}{6}\left[p^2\left(N^{-1}+3M^{-1}\right)\frac{\partial^4 W}{\partial t^4}-4p\left(3-2MN^{-1}\right)\Delta\frac{\partial^2 W}{\partial t^2}+\right.$$
$$\left.+8M\left(1-MN^{-1}\right)\Delta^2 W\right]=\Phi\left(\varphi_Z,\, f_{\iota z}\right) \quad (1)$$

Since the edges of $(y=0;l_2)$ the plate are hinged and supported, the solution of equation (1) will be sought in the form

$$W(x,y,t)=\exp\left(i\frac{b}{h}\xi t\right)\sum_{k=1}^{\infty}W_k(x)\sin\left(\frac{k\pi y}{l_2}\right) \quad (2)$$

Substituting (2) into equation (1), for W_k we obtain an ordinary differential equation

$$\frac{d^4W_k}{dx^4}+B_0\frac{d^2W_k}{dx^2}+B_1W_k=0 \quad (3)$$

Where the coefficients B_0, B_1 are equal

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$$B_0 = \left[\frac{A_1}{A_2} \xi^2 \left(\frac{b}{h} \right)^2 - 2 \left(\frac{k\pi}{l_2} \right)^2 \right];$$

$$B_1 = \left[\left(\frac{k\pi}{l_2} \right)^4 + \frac{A_0}{A_2} \xi^4 \left(\frac{b}{h} \right)^4 - \frac{A_1}{A_2} \xi^2 \left(\frac{b}{h} \right)^2 \left(\frac{k\pi}{l_2} \right)^2 - \frac{1}{A_2} \left(\frac{b}{h} \right)^2 \xi^2 \right] \quad (4)$$

The general solution of the equation (3) is written as

$$W_k(x) = C_1 \left[\frac{\cos(a_0 x)}{a_0^n} + \frac{\cos(a_1 x)}{a_1^n} \right] + C_2 \left[\frac{\cos(a_0 x)}{a_0^n} + \frac{\cos(a_1 x)}{a_1^n} \right] + + C_3 \left[\frac{\sin(a_0 x)}{a_0^m} + \frac{\sin(a_1 x)}{a_1^m} \right] +$$
$$+ C_4 \left[\frac{\sin(a_0 x)}{a_0^m} + \frac{\sin(a_1 x)}{a_1^m} \right], \quad (5)$$

where C $_j$ are the integration constants, the a $_i$,a $_j$ roots of the characteristic equation

$$a^4 + B_0 a^2 + B_1 = 0 \quad (6)$$

and equal

$$a_{0,1} = \sqrt{\frac{B_0}{2}} \pm \sqrt{\left(\frac{B_0}{2} \right)^2 - B_1} \quad (7)$$

The integers (n ,m) are chosen from the condition of simplifying the solution when the boundary condition on the left edge x= 0 is satisfied, and the other boundary conditions on X=l $_1$ lead to a transcendental equation for determining the self- frequencies of the plate.[3]

Let's consider some of the formulated tasks.

In this case, at the edges of the plate, we have the boundary conditions

$$W_k = \frac{dW_k}{dx} = 0; \qquad (x = 0;\ l_1) \quad (8)$$

Under boundary conditions (8) in the general solution (5), the numbers n=0; m=0, from the condition on the left end of the constant integration C $_1$, C $_2$ are zero, and from the conditions on the right end we get

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$$C_2\left[\cos(a_0 l_1)-\cos(a_1 l_2)\right]+C_4\left[\frac{\sin(a_0 l_1)}{a_0}-\frac{\sin(a_1 l_1)}{a_1}\right]=0$$

$$C_2\left[a_0\sin(a_0 l_1)-a_1\sin(a_1 l_1)\right]-C_4\left[\cos(a_0 l_1)-\cos(a_1 l_1)\right]=0 \quad (9)$$

whence from the condition of non-triviality of the solution we get the transcendental frequency equation

$$2-\frac{a_0^2+a_1^2}{a_0 a_1}\sin(a_0 l_1)\sin(a_1 l_1)-2\cos(a_0 l_1)\cos(a_1 l_1)=0 \quad (10)$$

Firstly, let us consider the simplest transcendental equation

$$\alpha_0\cos(\alpha_0 l_1)\sin(\alpha_1 l_1)-\alpha_1\sin(\alpha_0 l_1)\cos(\alpha_1 l_1)=0. \quad (11)$$

We introduce the notation

$$l=\frac{l_1}{h};$$

$$\alpha_{0.1}^{1}=\sqrt{\frac{B_0^1}{2}\pm\sqrt{\left(\frac{B_0^1}{2}\right)^2-B_1^1}};$$

$$B_0^{''}=\left[(2-\nu)\xi^2-2\gamma\right]; \qquad \gamma=\left(\frac{\pi k h}{l_2}\right)^2 \quad (12)$$

$$B_1^{;}=\left[\gamma^2+\frac{7-8\nu}{8}\xi^4-(2-\nu)\gamma\xi^2-\frac{3}{2}(1-\nu)\xi^2\right] \quad (12)$$

and we will omit the strokes in the future for simplicity. Since the sines and cosines of any argument are equal

$$\sin z=\sum_{i=0}^{\infty}(-1)^i\frac{z^{2i+1}}{(2i+1)!}$$

$$\cos z=\sum_{j=0}^{\infty}(-1)^j\frac{z^{2j}}{(2j)!};$$

That equation (11) is equivalent to the following

$$\alpha_0\alpha_1\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(-1)^{i+j}\frac{\alpha_1^{2i}\alpha_0^{2j}-\alpha_0^{2i}\alpha_1^{2j}}{(2i+1)!(2j)!}l^{2(i+j)}=0 \quad (13)$$

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If we accept the quantity a_0 is determined from the expression (7) with a plus sign under the root, then it follows this root does not vanish for any values of y, v, ξ .[4]

Therefore, at the beginning it is possible to $a_1 = 0$ or for ξ we obtain the equation

$$\xi^4 - \frac{8\left[(2-\nu)\gamma + \frac{3}{2}(1-\nu)\right]}{(7-8\nu)}\xi^2 + \frac{8\gamma^2}{(7-8\nu)} = 0; \quad (14)$$

whose roots are equal

$$\xi_{1,2} = (7-8\nu)^{-\frac{2}{2}}\sqrt{4\left[(2-\nu)\gamma + \frac{3}{2}(1-\nu)\right] \pm \sqrt{8(1+\nu^2)\gamma^2 + 3\gamma(1-\nu)(2-\nu) + \frac{9}{4}(1-\nu)^2}} \quad (15)$$

since the series in the expressions of trigonometric functions are convergent and the series in equation (12), equivalent to equation (11) is also convergent, in the study of the partial equation (13) it can be limited to a finite number of first terms.

Taking the first three terms in the series (3), we write it in the form

$$\alpha_0\alpha_1(\alpha_1^2 - \alpha_0^2)\left\{\frac{1}{3}l^2 - \frac{1}{30}(\alpha_1^2 + \alpha_0^2)l^4 + \right\}\left[\frac{1}{840}(\alpha_1^4 + \alpha_0^2\alpha_1^2 + \alpha_0^4) + \frac{1}{360}\alpha_0^2\alpha_1^2\right]l^6 + \dots \} = 0 \quad (16)$$

The roots of the expression α $_1$ = 0 are equal to (15). The quantity of $(a_1^2 - a_0^2)$ is non-zero for any values of y, v, ξ .

If in the expression (16) we take only the first two terms, we get

$$(\alpha_1^2 + \alpha_0^2) - 10l^{-2} = 0$$

Or

$$B_0 - 10l^{-2} = 0 \quad (17)$$

And frequency equation

$$\xi^2 = \frac{2\gamma + 10l^{-2}}{(2-\nu)}; \quad (18)$$

Positive root of which is equal to

$$\xi = \sqrt{\frac{2\gamma + 10l^{-2}}{(2-\nu)}} \quad (19)$$

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If we take all the first three terms in the expression, we get

$$\left[\left(\alpha_1^4+\alpha_0^4\right)+\frac{10}{3}\alpha_0^2\alpha_1^2\right]-28(\alpha_1^2+\alpha_0^2)l^{-2}+280l^{-4}=0 \qquad 20)$$

or

$$\left[B_0^2+\frac{4}{3}B_1\right]-28B_0l^{-2}+280l^{-4}=0$$

and their corresponding frequency equation

$$\left[(2-\nu)^2+\frac{7+8\nu}{6}\right]\xi^4-\left[(2-\nu)\left(\frac{16}{3}\gamma+28l^{-2}\right)+2(1-\nu)\right]\xi^2+$$

$$+\left[\frac{16}{3}\gamma^2+56\gamma l^{-2}+280l^{-4}\right]=0, \qquad (21)$$

which has two positive roots.

Similarly, one can take the first four or more terms in expression (13) and obtain a more accurate frequency equation and corresponding frequencies ξ .[5]

To find the frequency equation from the series of equation (13), it is necessary to clarify the condition of appropriate retention of a finite number of terms.

Let us apply the d'Alembert principle of series convergence to the series in equation (13). We get [6]

$$\left|\frac{\alpha_0^2\alpha_1^2l^2}{(2i+3)(2j+2)}\right|\le q^2<1 \qquad (22)$$

$0 < q < 1.$

where

From the inequality (22) implies that

$$\left|a_0^2a_1^2\right|\le q_{i,j}^2=q_{i,j}^2=q^2\,\frac{(2i+3)(2j+2)}{l^2} \qquad (23)$$

The analysis of inequality (23) shows that it is valid when the solving the inequality

$$-\left(\frac{8}{7-8v}\right)q_{i,j}^2\le\xi^4-2D\xi^2+E\le\left(\frac{8}{7-8v}\right)q_{i.j}^2=C_{i,j}^2$$

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where the coefficients *D, E* are equal

$$D = \frac{4\left[(2-v)\gamma + \frac{3}{2}(1-v)\right]}{(7-8v)}; \quad E = \frac{8\gamma^2}{(7-8v)}$$

or inequality

$D^2 - E \le C_{i,j}^2$ (24)

By the given parameters of a geometric and mechanical character from the inequality (24) one can determine the necessary number of first terms in series (13) for finding the frequency equation of relative frequencies ξ.

We consider the transcendental equation (10). Like transcendental equation (11), equation (10) is equivalent to the following

$$a_0 a_1 \left\{ 2\left[1 - \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (-1)^{i+j} \frac{a_0^{2i} a_1^{2j}}{(2i)!(2j)!} J^{2(i+j)}\right] - \right. \quad (25)$$

$$\left. -\left(a_0^2 + a_1^2\right) \sum_{i=0}^{\infty} \sum_{jj=0}^{\infty} (-1)^{i+j} \frac{a_0^{2i} a_1^{2j}}{(2i+1)!(2j+1)!} J^{2(i+j+1)} \right\} = 0$$

From the equation it follows that, at first, a $_1$ =0 and we get the frequencies (15).

We write equation (25) by writing the first terms

$$\left\{ (a_0^2 + a_1^2) J^2 - \frac{1}{6}(5a_0^4 + 5a_1^4 + a_0^2 a_1^2) J^4 + \right\} \frac{1}{90}\left[a_0^6 + a_1^6 + 7a_0^2 a_1^2 (a_0^2 a_1^2)\right] J^6 + \dots \} = 0$$

(26)

From (26) it also follows that we can assume $\left(a_0^2 + a_1^2\right) = 0$ and obtain

$B_0 = 0$

or

$\xi^2 - \frac{2\gamma}{(2-v)} = 0$ (27)

whose root is equal to

$\xi = \sqrt{\frac{2\gamma}{(2-v)}}$ (28)

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Likewise, we can suppose approximately

$$(a_0^2 + a_1^2) - \frac{J^2}{6}(5a_0^4 + 5a_1^4 + a_0^2 a_1^2) = 0 \quad (29)$$

and get the frequency equation

$$(5B_0^2 - 9B_1) - \frac{6}{J^2} B_0 = 0 \quad (30)$$

having positive roots.

Thus, transcendental frequency equations can be reduced to algebraic and to investigate the influence of both boundary conditions along the edges of a rectangular plate or a rectangular flat element, as well as geometric and mechanical character on the self **-** oscillatio n frequencies of rectangular flat elements .

Let us generalize the previous results for a rectangular plate or a flat element, the material of which satisfies the Maxwell viscoelastic model .[7]

Suppose we have a rectangular homogeneous isotropic plate.

In this case, the solution of an approximate fourth order equation will be sought in the form

$$W = \exp\left(\frac{b}{h}\xi t\right)\sum_{k=1}^{\infty} W_k \sin\left(\frac{pky}{J_2}\right), \quad (31)$$

where ξ is the complex frequency, the real part of which determines the law of damping of oscillations, and the imaginary part determines the frequencies of self-oscillations.

For W_k we get an ordinary differential equation

$$\frac{d^4 W_k}{dx^4} - \overline{B_0}\frac{d^2 W_k}{dx^2} + \overline{B_1} W_k = 0 \quad (32)$$

where $\overline{B_0}, \overline{B_1}$ are equal

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$$\overline{B_0} = \left[2\gamma + \frac{A_1}{A_2}\frac{b}{h}\left(\frac{b}{h}\xi^2 + \frac{1}{\tau}\xi \right) \right]; \quad (33)$$

$$\overline{B_1} = \left[\frac{A_0}{A_2}\left(\frac{b}{h}\right)^4 \xi^4 + 2\frac{A_0}{A_2\tau}\left(\frac{b}{h}\right)^3 \xi^3 + \left(\frac{b}{h}\right)^2 \left(\frac{1}{A_2} + \frac{1}{\tau^2}\frac{A_0}{A_2} + 2\gamma\frac{A_1}{A_2} \right)\xi^2 + \frac{1}{A_2\tau}\left(\frac{b}{h}\right) \times \right.$$
$$\left. \times \left(1 + 2\gamma \frac{A_1}{A_2} \right)\xi + \gamma^2 \right]$$

The coefficients A_j are given in the previous paragraphs.

The general solution of equation (32) we write in the form

$$W_k = C_1\left[\frac{ch(a_0 x)}{a_1^n} + \frac{ch(a_1 x)}{a_1^n} \right] + C_2\left[\frac{ch(a_0 x)}{a_0^n} + \frac{ch(a_1 x)}{a_1^n} \right] + \left[\frac{sh(a_0 x)}{a_0^m} + \frac{sh(a_1 x)}{a_1^m} \right] +$$
$$+ C_4\left[\frac{sh(a_0 x)}{a_0^m} + \frac{sh(a_1 x)}{a_1^m} \right],$$

(34)

that is instead of trigonometric functions, we have hyperbolic.

All the boundary value tasks considered above leading to transcendental equations are solved in a similar way. Transcendental equations are obtained from the previous ones, in which values a_j must be replaced by values ia_j, where i is an imaginary unit.[8]

For example, the transcendental equation (11) goes into the equation

$$a_0 ch(a_0 l_1) sh(a_1 l_1) - a_1 sh(a_0 l_1) ch(a_1 l_1) = 0 \quad (35)$$

which is equivalent to the following

$$a_0 a_1 \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{a_1^{2i} a_1^{2j} a_0^{2i}}{(2i+1)!(2j)!} l^{2(i+j)} = 0;$$

$$a_{0,1} = \sqrt{\frac{\overline{B_0}}{2} \pm \sqrt{\frac{\overline{B_0^2}}{4} - B_1}} \quad (36)$$

One of the frequency equations of the formula (36) follows from the condition $a_j = 0$ and we get

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$$\xi^4+\frac{2}{\tau_0}\xi^3+\frac{8}{(7-8v)}\left[(2-v)\gamma+\frac{(7-8v)}{8\tau_0^2}+\frac{3}{2}(1-v)\right]\xi^2+\frac{12(1-v)}{(7-8v)\tau_0}[1+2(2-v)\gamma]\xi+$$
$$+\frac{8}{(7-8v)}\gamma^2=0, \tag{37}$$

which coincides with the frequency equation for a rectangular hinged and supported plate on all four sides of the plate, has two complexly conjugate roots.[9]

3. METHODS

The theory of vibrations and the method for calculating the vibrations of a flat element, taking into account the above factors of a mechanical, rheological and geometric nature, are based on the consideration of a flat element in a three-dimensional setting of the mechanics of a solid deformed body, under the same boundary and initial conditions. The three-dimensional problem is solved using the methods of Fourier and Laplace integral transformations. In transformations, general solutions of a three-dimensional dynamic problem are constructed.

Let us present the results obtained by the above method for various types of flat element. The general equation of the plate oscillation with respect to the transverse displacement W of the points of the middle plane of the plate z = 0 has the form

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left\{\lambda_1^{(n)}\left[\lambda_2^{(1)}-\Delta\right]^2 DQ_m+\left[\lambda_2^{(1)}+\Delta\right]\lambda_1^{(n+m)}+4\lambda_2^{(1)}\Delta DQ_n\lambda_1^{(n)}\right\}W\frac{h^{2(n+m)+1}}{(2n+1)!(2m)!}=$$
$$=-\sum_{n=0}^{\infty}M^{-1}\left[\left(\lambda_2^{(1)}-\Delta\right)DQ_n+\lambda_1^{(n)}\right]f_z\frac{h^{2n}}{(2n)!}; \tag{38}$$

$$\sigma_{jz}^{\pm}=0;\quad f_z^{+}=-f_z^{-}=f_z;\quad D=1-MN^{-1},$$

where N and are M the viscoelastic operators

$$N(\zeta)=(\lambda+2\mu)\left[\zeta(t)-\int_0^{\infty}f_1(t-\xi)\zeta(\xi)d\xi\right],$$

$$M(\zeta)=\mu\left[\zeta(t)-\int_0^{\infty}f_1(t-\xi)\zeta(\xi)d\xi\right], \tag{39}$$

$f_z(x,y,t)$- external unsteady forces,

$\lambda_1^{(1)},\lambda_2^{(1)},Q_n$ operators of the form

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$$\lambda_1^{(1)} = \left[pN^{-1}\left(\frac{\partial^2}{\partial t^2}\right) - \Delta \right]; \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2};$$
$$\lambda_1^{(1)} = \left[pM^{-1}\left(\frac{\partial^2}{\partial t^2}\right) - \Delta \right]; Q_n = \sum_{q=0}^{n=1} \lambda_1^{(n-q-1)} \lambda_2^{2q} \quad (40)$$

The general equation (40) is complex in structure and is of little use for solving applied problems. From the general equation one can obtain approximate oscillation equations. For example, limiting ourselves in the series of sums of the equation to the first two terms, we obtain an approximate equation [10]

$$P_0(W) \equiv p\frac{\partial^2 W}{\partial t^2} + \frac{h^2}{6}\left[p^2\left(N^{-1} + 3M^{-1}\right)\frac{\partial^4 W}{\partial t^4} - 4p\left(3 - 2MN^{-1}\right)\Delta\frac{\partial^2 W}{\partial t^2} + \right. \quad (41)$$
$$\left. + 8M\left(1 - MN^{-1}\right)\Delta^2 W\right] = \frac{1}{h} f_z,$$

where $2h$ is the plate thickness. Equation (41) is a generalization of the Kirchhoff, S.P. Timoshenko and other authors.

If a homogeneous plate lies on a deformable base, then in equation (4 1) it is necessary to add the rebound law, which has the form

$$P_1(W) = \frac{S}{2h}\left\{\frac{\partial W}{\partial t} + \frac{h^2}{2}\frac{\partial}{\partial t}\left[p\left(M^{-1} + 3N^{-1}\right)\frac{\partial^2 W}{\partial t^2} - 4\Delta W\right]\right\};$$
$$S = M_1^{-1} M N_1^{1/2}\sqrt{p_1}; \quad (42)$$

M_1, N_1, p_1 base settings. As you can see, the repulse law is different from Winkler 's.

When solving various problems of vibration, say, of rectangular plates, it is necessary to formulate the boundary and initial conditions. The general solutions obtained and the dependences of displacements and stresses on the sought-for functions allow us to unambiguously and rigorously derive the boundary conditions. It is shown that for hinged and rigid fastening the boundary conditions coincide with the classical ones , and for a stress-free edge, the boundary conditions for a homogeneous isotropic plate of the form ($x = const$) are obtained:

$$(2 + 3D)\frac{\partial^2 W}{\partial x^2} + (1 + D)\frac{\partial^2 W}{\partial y^2} - p(1 + D)M^{-1}\left(\frac{\partial^2 W}{\partial t^2}\right) = 0;$$
$$\frac{\partial^3 W}{\partial x^3} = 0, \quad (43)$$

where one of the conditions contains an inertial component, which corresponds to the d'Alembert principle of mechanics. If the flat edge of the plate is in rigid contact with the deformable vertical plate, then the elastic embedment boundary condition has the form

$$\frac{h^3}{3}DM\frac{\partial^2 W}{\partial x^2}+\left[\frac{h^3}{3}DM-2h_1^2D_1M_1\right]\frac{\partial^2 W}{\partial y^2}=\left(\frac{h^2}{6}Dp-h_1^2D_1p_1\right)\frac{\partial^3 W}{\partial x\partial t^2};\ (44)$$

$$W=h_1\frac{\partial W}{\partial x};$$

where p_1, D_1, M_1 are the plate parameters.

4. CONCLUSION

Depending on the considered particular types of a flat element in the general solutions of a three-dimensional problem, the main unknown functions are chosen: displacements or deformations at points of a fixed plane of a flat element, in particular, in the middle plane of a plate of constant thickness. Displacements and stresses at an arbitrary point of a flat element are expressed in terms of the main unknown functions, which are determined from the boundary conditions on the surfaces of a flat element. The equations obtained for the main unknown functions and are the general equations for the vibration of a plane element, containing the derivatives of functions with respect to coordinates and time of any arbitrarily large order. General solutions are presented as power series over the thickness of a flat element. The general solution refers to an equation of the hyperbolic type, which describes the oscillatory and wave processes in a flat element. Restricting ourselves in the series of the general equation to a finite number of first terms, we obtain approximate equations for the vibration of one or another flat element.

Thus, the proposed approach makes it possible to rigorously construct approximate theories of vibrations of various types of flat elements.

Literature

1. Filippov I.G., S.I. Filippov, 1995. Dynamic stability theory of rods. Proceedings of the Russian-Polish seminar. Theoretical foundations of construction. Warsaw, pp.63-69.
2. Filippov I.G., S.I. Filippov, V.I. Kostin, 1995. Dynamics of two-dimensional composites. - Proceedings of the International Conference on Mechanics and Materials, USA, Los Angeles, pp.75-79.
3. Seitmuratov A., Yergalauova Z., Makhambayeva I., Bexeitova A. Axismetric problems of elastic layer oscillation limited by rigid or deformed boundries // News of the national academy of sciences of the republic of Kazakhstan-series of geology and technical sciences ISSN 2224-5278. -2018.-Vol. 1, no. 427, -pp.127-135
4. Seitmuratov A., Tileubay S., Toxanova S., Ibragimova N., Doszhanov B., Aitimov M.Zh . The problem of the oscillation of the elastic layer bounded by rigid bouhdaries // News of the national academy of sciences of the republic of Kazakhstan-series physico-mathematical -2018.-Vol. 5, N o 321, - pp.42 - 48 https://doi.org/10.32014/2018.2518-1726.6
5. Almagambetova A., Tileubay S., Taimuratova L., Seitmuratov A., Kanibaikyzy K. Problem on the distribution of the harmonic type relay wave // News of the national academy of sciences of the republic of Kazakhstan-series of geology and technical sciences . ISSN 2224-5278. -2019. –Vol. 1, no. 433 -pp. 242-247 https://doi.org/10.32014/2019.2518-170X.29

6. Ashirbayev N ., Ashirbayeva Zh., Sultanbek T., Shomanbayeva M. (2018) Waves of elastic stresses in the doubly connected domain . Bulletin of KarSU, Mathematics series. 2(90):18-25.
7. Seitmuratov A., Seylova Z., Kanibaikyzy K., Smakhanova A., Serikbol M. Approximate equation plate oscillation for transverse displacement of points of the median plane // News of the national academy of sciences of the republic of Kazakhstan-series of geology and technical sciences . ISSN 2224-5278-2018. –Vol.3, No. 429, -pp .258-265
8. Ashirbayev N.K., Banas J., Dubiel A. Solvability of an Integral Equation of Volterra - Wiener- Hopf Type// Abstract and Applied Analysis. – Volume 2014 (2014), Article ID 982079, 9 pages, DOI: 10.1155/2014/982079.
9. Medeubaev, N.K., Seytmuratov A. Zh., Ramazanov M.I. Solving Problems of Vibrational Processes of Isotropically Homogeneous Elastic Plates // Lobachevskii journal of mathematics ISSN: 1995-0802.-2020, Vol41, No. 9, -pp.1846-1853. DOI 10.1134/S1995080220090188
10. Seitmuratov A., Medeubaev N., Yeshmurat G., Kudebayeva G. Approximate solution of the an elastic layer vibration task being exposed of moving load // News of the national academy of sciences of the republic of Kazakhstan-series physico - mathematical . ISSN 1991-346 X - 2018. -Vol.2, No. 318, pp. 54 - 60