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GENERALIZED CESÀRO SEQUENCE SPACE OF NON-ABSOLUTE AND ABSOLUTE TYPE AS COMPLETE PARANORMED SPACE

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Abstract

This work aims to introduce and study a new class of generalized Cesàro sequence space of non-absolute and absolute type .. Besides the investigation of the linear structures of the classes Ces(P) and Ces_P, our primary interest is to explore the paranormed structures and completeness of the classes Ces(P) and Ces_P when topologized with suitable Natural Paranorms.

Keywords and Phrases: Paranormed Space, Sequence Space, Cesàro Sequence Space.

2020 Mathematics Subject Classification: 40A05, 40C05, 46A45, 46B45.

1. INTRODUCTION

Before proceeding with the work, we recall some of the basic notations and definitions that are used in this paper.

A sequence space is defined to be a linear space of scalar (real or complex) functions on a countable set. The study of sequence spaces is thus a special case of the more general study of function spaces. The classical sequence spaces or scalar-valued sequence spaces (real or complex) have proven their worth as a big contribution in introducing the spaces of Cesàro almost convergent sequences (Kuddusi, and Şengönül [8]). The matrix transformations involving the notation of almost convergence are also considered in various mathematical researches. Thus the vector-valued sequence spaces, are the natural generalizations of the scalar-valued sequence spaces where the sequence spaces are those of vectors from some vector spaces.

Several workers like Altin and Tripathy [2], Kadak [5], Khan [6], Kolk [7], Maddox [9], Pahari [13], Ruckle [16], etc. have made their contributions and enriched the theory. Today, a decisive break occurred with the theory of scalar-valued sequences when

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considering the action of infinite matrices of linear operators from a space into another on sequences of the elements, (Sarigol and M.Mursaleen, [18]).

2. CLASSICAL SEQUENCE SPACES

Let $x = (x_1, x_2, x_3, ...)$, $y = (y_1, y_2, y_3,...)$ be two sequences (or points) with terms (or co-ordinates) $x_1, x_2, x_3,....$ and $y_1, y_2, y_3,....$ respectively where x_i's and y_i's are complex numbers. Let us write

$x + y = (x_1 + y_1, x_2 + y_2, x_3 + y_3,....)$ and $\alpha x = (\alpha x_1, \alpha x_2, \alpha x_3, ...)$, where $\alpha \in$ | is a complex number. Let ω be the collection of all sequences of complex numbers and $x, y, z, \ldots \in \omega$ and $\alpha, \beta \in$ | then

$$x + y \in \omega, \alpha x \in \omega.$$

If $p = (p_k)$ be a sequence of positive numbers (not necessarily bounded in general), X denotes the normed space, and x_k the elements of X. Then we have the following well-known sequence spaces.

i. $\ell_\infty = \{x = (x_k) \in \omega : \sup_{k\geq 1} |x_k| < \infty\}$, known as the space of bounded complex sequences;

ii. $\ell_\infty(p) = \{x = (x_k) \in \omega : \sup_{k\geq 1} |x_k|^{p_k} < \infty\}$;

iii. $\ell_\infty(X) = \{\overline{x} = (x_k): x_k \in X, \sup_{k\geq 1} \|x_k\| < \infty\}$.

$\ell_\infty(p)$ and $\ell_\infty(X)$ appear in the work of Maddox [9] and others.

Cesàro sequence spaces of absolute type were defined and studied in the work of Roopaei & Başar [15] and Saejung [17]. Ahmad & Mursaleen [1] in defined Cesàro sequence spaces of a bounded type. Nanda and Mohanty [10] have studied paranormed Cesàro sequence spaces. Ahmad & Mursaleen [1] introduced the new concept of an almost convergent sequence. A bounded sequence (x_k) of complex numbers is almost convergent if the sequence (t_{nk}) converges uniformly in n as $k \to \infty$, where

$$t_{nk} = \frac{1}{k}\sum_{i=1}^{k} x_{n+i}, \; n \in \mathbb{Z}^+.$$

Definition:Let X be a linear space over the field | or | with zero element θ. A mapping $G: X \to$ | is called paranorm if it satisfies the following conditions

$PN1$: $G(\theta) = 0$

$PN2$: $G(x) = G(-x)$

$PN3$: $G(x + y) \leq G(x) + G(y)$

$PN4$: If (α_n) is a sequence of scalars with $(\alpha_n) \to \alpha$ as $n \to \infty$ and (x_n) is a sequence in X with

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$G(x_n - x) \to 0$ as $n \to \infty$ then $G(\alpha_n x_n - \alpha x) \to 0$ as $n \to \infty$ (*continuity of scalar multiplication*)

The paranorm is called **total** if in addition, we have

$PN5$: $G(x) = 0$ implies $x = \theta$.

The concept of paranorm is closely related to linear metric space, (see, Wilansky [22]) and its studies on sequence spaces were initiated by Maddox [9] and many others. In particular, various types of paranormed sequence spaces were further studied by several workers, for instances we refer to a few: Ghimire & Pahari [4], Pahari [12], [13], Paudel, Pahari & Kumar [14], Nath & Tripathy [11]. Srivastava & Pahari [19], Tripathy [20], Tripathy & Sarma [21] studied the various types of topological structures of vector-valued sequence spaces defined by Orlicz function endowed with suitable natural paranorms and extended some of them in 2-normed spaces.

A paranormed space is a pair (X, G), where X is a vector space and G is a paranorm on X. Every paranormed space becomes a linear pseudo metric space on setting $d(x, y) = G(x - y)$. Thus each paranormed space is a topological vector space. The paranormed space (X, G) is called complete if (X, d) is complete.

3. THE CLASS Ces(*P*) and Ces$_P$

Let ω denote the vector space of all complex-valued sequences $x = (x_k)$. For a fixed positive number n and $0 < p < 1$, Ahmad & Mursaleen [1] constructed the following Cesàro sequence space of bounded type

$$S(p) = \textbf{Error!} \qquad \ldots(3.1)$$

Nanda & Mohanty [10] studied the p-norm on $S(p)$ defined as follow:

$$\|x\| = \sup_{\substack{k\geq 1 \\ n\geq 0}} \textbf{Error!},\ x \in S(p) \qquad \ldots(3.2)$$

Let $P = (P_k)$ be a bounded sequence of strictly positive real numbers and $0 < P_k$ □ M and let X be the vector space over the field ∣ of complex numbers. We throughout take co-ordinatewise vector operations i.e. $x + y = (x_k + y_k)$ and □x = (□x_k).

Let $M = \max \{1, \sup P_k\}$. We now construct the following classes of generalized Cesàro sequence space of non-absolute and absolute type denoted by Ces(P) and Ces$_P$, respectively as follows:

$$\text{Ces}(P) = \textbf{Error!}, \qquad \ldots(3.3)$$

$$\text{Ces}_P = \textbf{Error!} \qquad \ldots(3.4)$$

4. MAIN RESULTS

In this section, we shall investigate some results that characterize the linear topological structure of the classes Ces (P) and Ces$_P$ by endowing them with suitable natural paranorm. We shall use frequently: $A(\alpha) = \max \{1, |\alpha|\}$..

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Theorem 4.1. *The class Ces(P) is a linear paranormed space with paranorm G defined by*

$G(x) = \sup_{k\geq 1}$ **Error!**.

Proof : For any complex number α and $x = (x_k)$, $y = (y_k) \in$ Ces(P), we have

$$\sup_{k\geq 1} \frac{1}{k}\sum_{i=1}^{k} |x_i + y_i|^{Pk/M} \leq \sup_{k\geq 1} \frac{1}{k}\sum_{i=1}^{k} |x_i|^{Pk/M} + \sup_{k\geq 1} \frac{1}{k}\sum_{i=1}^{k} |y_i|^{Pk/M} < \infty$$

and $$\sup_{k\geq 1} \frac{1}{k}\sum_{i=1}^{k} |\alpha x_i|^{Pk/M} \leq \sup_{k\geq 1} |\alpha|^{Pk/M} \sup_{k\geq 1} \frac{1}{k}\sum_{i=1}^{k} |x_i|^{Pk/M}$$

$$\leq A(\alpha) \sup_{k\geq 1} \frac{1}{k}\sum_{i=1}^{k} |x_i|^{Pk/M} < \infty$$, where $A(\alpha) = \max\{1, |\alpha|\}$

It follows from the above inequalities that Ces(P) is a linear space with respect to point - wise linear operations. To prove paranormed space, we have

PN$_1$**:** $G(\theta) = \sup_{k\geq 1}$ **Error!** = 0.

PN$_2$**:** $G(x) = \sup_{k\geq 1}$ **Error!** = **Error! Error!** = $G(-x)$

PN$_3$**:** $G(x + y) = \sup_{k\geq 1}$ **Error!** $\leq$ **Error! Error!**+ **Error! Error!**= $G(x) + G(y)$

PN$_4$**:** The scalar multiplication is continuous in Ces(P)

Let a sequence (x^n) converges to x and (α_n) be a sequence of scalars that converges to scalar α, i.e.$(x^n) \to x$ as $n \to \infty$. Let $\varepsilon > 0$ be given, then there exists $N_1 \in \mathbb{Z}^+$ such that

$d(x^n, x) < \frac{\varepsilon}{3}$

$\Rightarrow \sup_{k}$ **Error!** < **Error!**, $\forall\ n > N_1$

And since $\alpha_n \to \alpha$ as $n \to \infty$. There exists $N_2 \in \mathbb{Z}^+$ such that

$d(\alpha_n, \alpha) < \frac{\varepsilon}{3}$, $\forall\ n > N_2$

$\Rightarrow |\ \alpha_n - \alpha|^{Pk/M} < \frac{\varepsilon}{3}$

Let $N_0 = \max\{N_1, N_2\}$, then $\forall\ n \geq N_0$, we have

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$d(\alpha_n x^n, \alpha x)$

$= \sup_{k\geq 1}$ **Error!**

$= \sup_{k\geq 1}$ **Error!**

$\leq \sup_{k\geq 1} \frac{1}{k} \sum_{i=1}^{k} [|\alpha_n - \alpha|^{Pk/M} |x_i^n - x_i|^{Pk/M} + |\alpha_n - \alpha|^{Pk/M} |x_i|^{Pk/M} + \quad |\alpha|^{Pk/M} |x_i^n - x_i|^{Pk/M}]$

$\leq \sup_{k\geq 1} \frac{1}{k} \sum_{i=1}^{k} |\alpha_n - \alpha|^{Pk/M} |x_i^n - x_i|^{Pk/M} + \sup_{k\geq 1} \frac{1}{k} \sum_{i=1}^{k} |\alpha_n - \alpha|^{Pk/M} |x_i|^{Pk/M} + \sup_{k\geq 1} \frac{1}{k} \sum_{i=1}^{k} |\alpha|^{Pk/M} |x_i^n - x_i|^{Pk/M}$

or, $d(\alpha_n x^n, \alpha x) \leq \Sigma_1 + \Sigma_2 + \Sigma_3$ …(4.1)

Where, $\Sigma_1 = \sup_{k\geq 1}$ **Error!** $\leq$ **Error!** < **Error!** …(4.2)

$\Sigma_2 = \sup_{k\geq 1}$ **Error!**

$\leq \frac{\varepsilon}{3M_1} \cdot \sup_{k\geq 1}$ **Error!** , where M_1 = **Error! Error!** $< \infty$ is used

$< \frac{\varepsilon}{3M_1} \cdot M_1 = \frac{\varepsilon}{3}$ …(4.3)

and, $\Sigma_3 = \sup_{k\geq 1}$ **Error!** $< M_0$. **Error!** = **Error!** …(4.4)

where $|\alpha|^{Pk/M} = \max\{1, |\alpha|\} = M_0$ is used.

Then in view of (4.2), (4.3) and (4.4), the relation (4.1) becomes

$d(\alpha_n x^n, \alpha x) < \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon, \forall\, n \geq N_0$

or, $d(\alpha_n x^n, \alpha x) < \varepsilon, \forall\, n \geq N_0$.

Therefore scalar multiplication is continuous at each point (α, x). In view of PN_1, PN_2, PN_3 ,and PN_4 , we conclude that Ces(P) is a paranormed space. This completes the proof.

Theorem 4.2. The paranormed metric space Ces(P) is complete.

Proof: We show that Ces(P) is complete with respect to the metric $d(x, y) = G(x - y)$.

Let (x^n) be a Cauchy sequence in Ces(P) and let $0 < \varepsilon < 1$ be given. Then there exists a $N_0 \in \mathbb{Z}^+$ such that

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$d(x^n, x^m) < \varepsilon, \forall n, m \geq N_0$

$\Rightarrow G(x^n - x^m) < \varepsilon, \forall n, m \geq N_0$

$\Rightarrow \sup_{k\geq 1}$ **Error!** $< \varepsilon, \forall n, m \geq N_0$ …(4.5)

or, **Error!** $< \varepsilon, \forall n, m \geq N_0$ & $\forall k \in \mathbb{Z}^+$

or, $|x_i^n - x_i^m| < \varepsilon^{M/Pk} < \varepsilon$, , $\forall n, m \geq N_0$ & $\forall k \in \mathbb{Z}^+$.

This shows that (x_i^n) is a Cauchy sequence in |. But | is complete, so there exists $x_i \in$ | for each i such that $x_i^n \to x_i$ as $n \to \infty$.

Define $x = (x_i)_{i=1}^{\infty}$ and taking limit $m \to \infty$ in (4.5), we get

$\sup_{k\geq 1}$ **Error!** $< \varepsilon, \forall n \geq N_0$ …(4.6)

$\Rightarrow G(x^n - x) < \varepsilon, \forall n \geq N_0$

or, $d(x^n, x) < \varepsilon, \forall n \geq N_0$

or, $x^n \to x$ as $n \to \infty$.

We show that x actually lies in Ces(P).

Now by triangle inequality, and in view of (4.6) ,we have

$d(x, \theta) \leq d(x, x^n) + d(x^n, \theta)$

or, $G(x - \theta) \leq G(x - x^n) + G(x^n - \theta)$

or, $\sup_{k\geq 1}$ **Error!** $\leq$ **Error! Error!** + **Error! Error!**

or, $\sup_{k\geq 1}$ **Error!** $\leq \varepsilon$ + finite number $< \infty$

Thus $\sup_{k\geq 1}$ **Error!** $< \infty$.

This shows that $x = (x_i)$ = Ces(P) and hence Ces(P) is a complete paranormed space.

This completes the proof.

Theorem 4.3 The class Ces_P = **Error!** is a linear metric space.

The proof is omitted as it can be proved in accordance with the proof of the Theorem 4.1.

Theorem 4.4 The linear metric space Ces_P is a paranormed by H defined

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$H(x) = \sup_{k\geq 1}$ **Error!**, where $M = \max\{1, \sup P_k\}$.

Proof : Clearly, PN_1 : $H(\theta) = 0$, PN_2 : $H(x) = H(-x)$, PN_3 : $H(x + y) \leq H(x) + H(y)$.

For PN_4 : The scalar multiplication is continuous in Ces_P

Let (x^n) be a sequence converges to x and (α_n) be a sequence of scalars that converges to a scalar α, i.e. $(x^n) \to x$ as $n \to \infty$. Let $\varepsilon > 0$ be given. Then there exists a $N_1 \in \mathbb{Z}^+$ such that

$d(x^n, x) < \frac{\varepsilon}{3}, \forall\ n \geq N_1$

$\Rightarrow \sup_{k\geq 1}$ **Error!** < **Error!** < **Error!**, for all $n \geq N_1$,where $\sup\{1, |\lambda|^{Pk/M}\} = M_0$ is used.

Also, $(\alpha_n) \to \alpha$ as $n \to \infty$. Let $\varepsilon > 0$ be given, then there exists $N_2 \in \mathbb{Z}^+$ such that

$d(\alpha_n, \alpha) < \varepsilon < 1, \forall\ n \geq N_2$

or, $|\alpha_n - \alpha| < 1$

or, $\left(|\alpha_n - \alpha|^{Pk}\right)^{1/M} < 1, \ \forall\ n \geq N_2$

Let $N_0 = \max\{N_1, N_2\}$. Then $\forall\ n \geq N_0$, we have

$d(\alpha_n x^n, \alpha x) = H(\alpha_n x^n - \alpha x)$

$= \sup_{k\geq 1}$ **Error!**

$= \sup_{k\geq 1}$ **Error!**

$\leq \sup_{k\geq 1}$ **Error!**+ **Error! Error!** + **Error! Error!**

or, $d(\alpha_n x^n, \alpha x) = \Sigma_1 + \Sigma_2 + \Sigma_3$ …(4.7)

Now, $\Sigma_1 = \sup_{k\geq 1}$ **Error!** …(4.8)

$\leq \sup_{k\geq 1} |\alpha_n - \alpha|^{Pk/M}$ **Error!**

$< 1. \sup_{k\geq 1}$ **Error!**[Using $|\alpha_n - \alpha|^{Pk/M} < 1$]

$< \frac{\varepsilon}{3\ M_0} < \frac{\varepsilon}{3}, \forall\ n \geq N_1$

$\Sigma_2 = \sup_{k\geq 1}$ **Error!** …(4.9)

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$\leq |\alpha_n - \alpha|^{Pk/M} \sup_{k\geq 1}$ **Error!**

$< \frac{\varepsilon}{3M_1} . M_1 < \frac{\varepsilon}{3}$ **Error!**

and $\Sigma_3 = \sup_{k\geq 1}$ **Error!** ...(4.10)

$\leq \sup_{k\geq 1} |\alpha|^{Pk/M}$ **Error!**

$< M_2 . \frac{\varepsilon}{3M_2} = \frac{\varepsilon}{3}$, where $|\alpha|^{Pk/M} = \max\{1,|\alpha|\} = M_2$ is used.

Using (4.8), (4.9) and (4.10) in (4.7), we have

$d(\alpha_n x^n, \alpha x) < \varepsilon, \quad \forall\ n \geq N_0$

Thus scalar multiplication is continuous at each point (α, x).

Theorem 4.5 The linear metric space Ces_P is complete.

Proof: We show that Ces_P is complete with respect to the metric $d(x, y) = H(x - y)$.

Let (x^n) be a Cauchy sequence in Ces_P and $0 < \varepsilon < 1$ be given. Then there exists $N_0 \in \mathbb{Z}^+$ such that

$d(x, y) = H(x^n - x^m) < \varepsilon, \forall\ n, m \geq N_0$

or, $\sup_{k\geq 1}$ **Error!** $< \varepsilon, \forall\ n, m \geq N_0$ ...(3.11)

or, **Error!** $< \varepsilon, \forall\ n, m \geq N_0$ and $\forall\ k \in \mathbb{Z}^+$.

or, $|x_i^n - x_i^m| < \varepsilon^{M/Pk} < \varepsilon, \forall\ n, m \geq N_0$ and $\forall\ k \in \mathbb{Z}^+$.

This shows that for a fixed $i \in \mathbb{Z}^+$, the sequence (x_i^n) is a Cauchy sequence in complex numbers |.

Since | is complete, therefore there exists $x_i \in$ | such that $(x_i^n)_{n=1}^{\infty}$ converges to x_i.

Let $x_i^n \to x_i$ as $n \to \infty$ for each $i \in \mathbb{Z}^+$.

Define $x = (x_i)_{i=1}^{\infty}$ and taking limit $m \to \infty$ in (3.11), we get

$\sup_{k\geq 1}$ **Error!** $< \varepsilon, \forall\ n \geq N_0$ ...(3.12)

This shows that $H(x^n - x) < \varepsilon, \forall\ n \geq N_0$ and hence $x^n \to x$ as $n \to \infty$.

Now we show that x actually lies in Ces_P.

$H(x - \theta) \leq H(x - x^n) + H(x^n - \theta)$

or, $\sup_{k\geq 1}$ **Error!** $\leq$ **Error! Error!** + **Error! Error!**

$< \varepsilon +$ finite number $< \infty$

Therefore, $\sup_{k\geq 1}$ **Error!** $< \infty$ and hence $x = (x_i) \in \text{Ces}_P$

Therefore, Ces_P is a complete paranormed space. This completes the proof.

5. CONCLUSION

In this work, we have introduced and studied a new class of generalized Cesàro sequence space of non-absolute and absolute type.. We have explored the linear structures of the classes Ces(P) and and Ces_P when topologized them with suitable natural paranorms. In fact, these results can be used in the fields of Functional Analysis, Fourier series, and Engineering to investigate other properties of the infinite series and sequences.

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Data Availability

All the data used in this study "Generalized Cesàro Sequence Space of Non-Absolute and Absolute type as Complete Paranormed Space" supports the findings and are cited within the article.

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