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SAROJ VEENA'S MATHEMATICAL MODEL TO RECOMMEND THE BEHAVIOUR OF AN IDEALIZED VIBRATING STRING

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Abstract

The goal of this research is to establish a mathematical model for prescribing the behaviour of a Saroj Veena idealised vibrating string. We can measure theoretical amplitudes for the various harmonics using the mathematical model, depending on the magnitude and location of the pluck. An inquiry into the effect of the position and method of plucking a Saroj Veena string on the distribution of energy between simple tone and overtones is addressed in this article. It was accomplished by collecting sound recordings from a Saroj Veena, and by using computer tools to interpret the data numerically. In conclusion, it is mentioned that when the string was plucked close to the edge of the Saroj Veena, there were more overtones made. Since the paper was an analysis of physics, it experimentally investigated the matter, offering few or no mathematical reasons for the findings.

Keywords: Saroj Veena, Harmonics, Amplitudes, Displacement functions, Fourier series.

1. Introduction

Recent years of physical model-based synthesis have seen remarkable progress. Learning the material science and acoustics of the instruments in these tests is a potential starting point for the show. The models can be computationally qualified by some disentanglement and they can then be modified to keep running on a PC continuously. Since actualized physical representations are derived from the instrument's material science, they bring about the amalgamation of particularly realistic instrumental sounds. In any case, where it is suggested that the graphical model running on a Computer be played, then analysis must be applied to the behaviour of the entertainer to see how to interact with the model of the Computer. There are efficient string amalgamation formulas for the particular example of the guitar, and they are continually being developed (D.A Jaffe 1983) (M. Karjalainen 1993) (Smith 1993) (V. Valimaki 1996) (M.Karjalainen 1998). Analysis aims to explain the relations between timbre subtleties and model (T. Tolonen 1997), tactile, verbal (C.Erkut 2000) (G.Cuzzucoli 1999) and psycho acoustical (N.Orio 1999) criteria were introduced for the inquiry partner. The culling point location on the string has a remarkable effect on the timbre subtlety (O. S. Caroline Traube 2000) among the parameters that can be omitted. Fingering the left hand is also urgent. There are special ways to finger harmonies or conducting music, truth is told. A specific fingering would be selected

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because it is perfect, productive and easy to carry, or because it sounds in a particular and imagined manner. It is easy to play a few tones on a guitar with up to five different strings / fret blends. In this sense, whether the key data available is a video, the fingering used by a single entertainer is not normally noticeable or apparent. The region of the culling point along the string has a remarkable influence, among the instrumental motion parameters that contribute to the timbre of a guitar sound (P. D. Caroline Traube 2003). Culling a string across the scaffold provides a sound that is cleaner, stronger and more pronounced in volume. The tone in elevated recurrence segments is richer. When playing the sulponticello guitar (P. D. Caroline Traube 2003), this arises. Sultasto (P. D. Caroline Traube 2003), near or over the fingerboard, closer to the midpoint of the loop, is the other incredible one. All things considering, in high recurrence pieces, the sound is quieter, mellower, and less rich. Behind the sound opening is actually the nonpartisan location of the right side. The low strings are normally culled farther away from the scaffold than the higher ones (O. S. Caroline Traube 2000) in view of the position of the right hand fingers.

1.1 Motivation

To try to explain the disclosures, using a computational approach, was one of the key inspirations for this article. The goal of the study is to develop a computer model that predicts how a vibrating string will behave. Such simplification assumptions should be linked to the science model in order to keep it logical. Towards the end of the expose, a few clarifications of the model are presented.

2. RELATED WORK

The developer (O. S. Caroline Traube 2000) suggested a recurrence space technique to determine the culling point from an acoustically registered sign on a guitar string. In addition, it implements, in view of the culling point details, a specific method for defining the fingering point. In (P.D. Caroline Traube 2003) the developer recommended an iterative calculation of the basic parameters of the ghostly envelope on the extraction of the excitation point field on a guitar string. This paper creator suggested a general methodology to determine the area of the culling point, operating in two stages: a weighted slightest square approximation is used to optimize a FIR brush channel deferral value to best match the deliberate ghostly envelope, starting with a calculation defined with the autocorrelation of the sign as a first rough guess. In (C.Erkut 2000), the developer suggested that model parameter modification is a crucial sub-issue for rational model-based sound union. This paper discusses the alteration of an existing guitar show's adjustment mechanism, and expands the technique of parameter extraction to collect data on execution attributes, such as damping administrations, rehashed culls, vibrato values, distinctive types of fearlessness and varieties of components.

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3. MATHEMATICAL MODEL

The general structure of the mathematical assertion of a one-dimensional wave, with both ends defined (Linhart n.d), is:

$$y(x,t) = \sum_i X_i(x)T_i(t) \; = \sum_{k=1}^{\infty} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \cos\left(\frac{ck\pi}{L}t\right) + \beta_k \sin\left(\frac{ck\pi}{L}t\right)\right) \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Where,

$c=\sqrt{T/\mu}$

The natural frequencies are $\frac{ck\pi}{L}$

3.1 Imposing initial conditions

When plucking the line, it is separated from its equilibrium state by the distance (h) at position d. A function f(x) is defined by the form of a string the moment it is plucked.

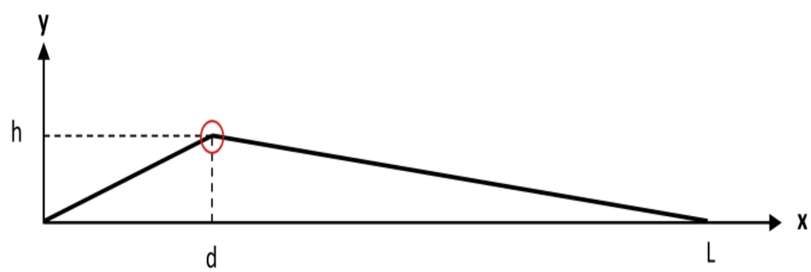

Figure: Initial conditions of the plucked string.

Since we assume that when released (no speed), the string is motionless, the original conditions are:

$$y(x,0) = f(x) \qquad \text{for all } 0 < x < L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\frac{\partial}{\partial t}y(x,0) = 0 \qquad \text{for all } 0 < x < L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

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Using (1) gives:

$$f(x) = y(x,0) = \sum_{k=1}^{\infty} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \cos\left(\frac{ck\pi}{L} \times 0\right) + \beta_k \sin\left(\frac{ck\pi}{L} \times 0\right)\right)$$

$$= \sum_{k=1}^{\infty} \alpha_k \sin\left(\frac{k\pi}{L}x\right) \qquad \text{......... (4)}$$

$$\frac{\partial}{\partial t} y(x,0) = \frac{\partial}{\partial t} \sum_{k=1} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \cos\left(\frac{ck\pi}{L}t\right) + \beta_k \sin\left(\frac{ck\pi}{L}t\right)\right)$$

$$= \sum_{k=1}^{\infty} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \frac{\partial}{\partial t}\cos\left(\frac{ck\pi}{L}t\right) + \beta_k \frac{\partial}{\partial t}\sin\left(\frac{ck\pi}{L}t\right)\right)$$

And

$$\frac{\partial}{\partial t} y(x,0) = \frac{\partial}{\partial t} \sum_{k=1} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \cos\left(\frac{ck\pi}{L}t\right) + \beta_k \sin\left(\frac{ck\pi}{L}t\right)\right)$$

$$= \sum_{k=1}^{\infty} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \frac{\partial}{\partial t}\cos\left(\frac{ck\pi}{L}t\right) + \beta_k \frac{\partial}{\partial t}\sin\left(\frac{ck\pi}{L}t\right)\right)$$

Using the chain rule for derivatives:

$$\frac{\partial}{\partial t}\cos\left(\frac{ck\pi}{L}t\right) = -\left(\frac{ck\pi}{L}\right)\sin\left(\frac{ck\pi}{L}t\right)$$

$$\frac{\partial}{\partial t}\sin\left(\frac{ck\pi}{L}t\right) = \left(\frac{ck\pi}{L}\right)\cos\left(\frac{ck\pi}{L}t\right)$$

At t=0:

$$\left.\frac{\partial}{\partial t}\cos\left(\frac{ck\pi}{L}t\right)\right|_{t=0} = -\left(\frac{ck\pi}{L}\right)\sin\left(\frac{ck\pi}{L} \times 0\right) = 0$$

$$\left.\frac{\partial}{\partial t}\sin\left(\frac{ck\pi}{L}t\right)\right|_{t=0} = \left(\frac{ck\pi}{L}\right)\cos\left(\frac{ck\pi}{L} \times 0\right) = \left(\frac{ck\pi}{L}\right)$$

Giving:

From (3):

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$$\frac{\partial}{\partial t}y(x,0) = 0$$

$$\sum_{k=1}^{\infty} \beta_k \frac{ck\pi}{L} \sin\left(\frac{k\pi}{L}x\right) = 0$$

$$\frac{\partial}{\partial t}y(x,0) = \sum_{k=1}^{\infty} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \times 0 + \beta_k \frac{ck\pi}{L} 1\right) = \sum_{k=1}^{\infty} \beta_k \frac{ck\pi}{L} \sin\left(\frac{k\pi}{L}x\right)$$

Implying $\beta_{k=0}$ for all k. This allows us to simplify (1) to:

$$y(x,t) = \sum_{k=1}^{\infty} \alpha_k \sin\left(\frac{k\pi}{L}x\right) \cos\left(\frac{ck\pi}{L}t\right) \qquad \ldots\ldots\ldots\ldots.. (5)$$

3.2 Fourier series

Any smooth function f(x) has a unique representation

$$f(x) = \sum_{k=1}^{\infty} A_k \sin\left(\frac{k\pi}{L}x\right)$$

Where the coefficients are computed by

$$A_k = \frac{2}{L}\int_0^L f(x) \sin\left(\frac{k\pi x}{L}\right) dx \qquad \ldots\ldots\ldots\ldots\ldots.. (6)$$

To find the coefficients α_k in (5) Fourier series will be used.

As noted earlier, the shape of the string at the moment it is plucked can be defined by a function f(x).

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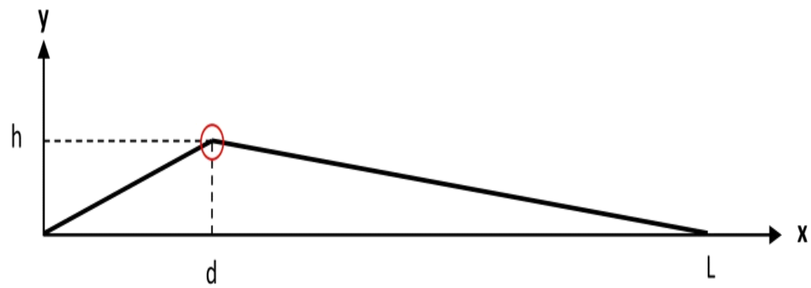

$$f(x) = \begin{cases} \dfrac{hx}{d}, & 0 \le x \le d \\ \dfrac{h(L-x)}{L-d}, & d < x \le L \end{cases}$$

Equation (4) takes precisely the form of a sequence of Fouriers. The coefficients are determined by (6), calculating the integral separately from 0 to d and from d to L. Using the form integral by sections, resolving the integral is completed.

Integration by parts (Wikipedia) (encyclopedia n.d.):

$$\int_a^b u\, dv = [uv]_a^b - \int_a^b v\, du$$

Coefficients in (4):

$$A_k = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{k\pi x}{L}\right)dx = \frac{2}{L}\left[\underbrace{\int_0^d \frac{hx}{d}\sin\left(\frac{k\pi x}{L}\right)dx}_{\text{Part 1}} + \underbrace{\int_d^L \frac{h(L-x)}{L-d}\sin\left(\frac{k\pi x}{L}\right)dx}_{\text{Part 2}}\right]$$

Part: 1

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Setting $u = \frac{hx}{d}$ and $dv = \sin\left(\frac{k\pi x}{L}\right)dx$

$$\int_0^d \frac{hx}{d}\sin\left(\frac{k\pi x}{L}\right)dx = \left[\frac{hx}{d}\left(\frac{-L}{k\pi}\right)cos\left(\frac{k\pi x}{L}\right)\right]_0^d - \int_0^d \left(\frac{-L}{k\pi}\right)\cos\left(\frac{k\pi x}{L}\right)\frac{h}{d}\,dx$$

$$= \left[\frac{-hLx}{k\pi d}cos\left(\frac{k\pi x}{L}\right)\right]_0^d - \int_0^d \frac{-hL}{k\pi d}\cos\left(\frac{k\pi x}{L}\right)dx$$

$$= \left[\frac{-hLx}{k\pi d}cos\left(\frac{k\pi x}{L}\right)\right]_0^d - \left(\frac{-hL}{k\pi d}\right)\left[\frac{L}{k\pi}\sin\left(\frac{k\pi x}{L}\right)\right]_0^d$$

$$= \frac{-hLd}{k\pi d}cos\left(\frac{k\pi d}{L}\right) - \left(\frac{-hL\times 0}{k\pi d}\right)cos\left(\frac{k\pi\times 0}{L}\right)$$

$$+\frac{hL}{k\pi d}\,\frac{L}{k\pi}\left(\sin\left(\frac{k\pi d}{L}\right) - \sin\left(\frac{k\pi\times 0}{L}\right)\right)$$

$$= \frac{-hL}{k\pi}cos\left(\frac{k\pi d}{L}\right) + \frac{hL^2}{k^2\pi^2 d}\sin\left(\frac{k\pi d}{L}\right)$$

Part: 2

Setting $u = \frac{h(L-x)}{L-d}$ and $dv = \sin\left(\frac{k\pi x}{L}\right)dx$

$$\int_d^L \frac{h(L-x)}{L-d}\sin\left(\frac{k\pi x}{L}\right)dx = \left[\frac{h(L-x)}{L-d}\left(\frac{-L}{k\pi}\right)cos\left(\frac{k\pi x}{L}\right)\right]_d^L - \int_d^L \left(\frac{-L}{k\pi}\right)\cos\left(\frac{k\pi x}{L}\right)\frac{-h}{L-d}\,dx$$

$$= \left[\frac{-h(L-x)L}{(L-d)k\pi}cos\left(\frac{k\pi x}{L}\right)\right]_d^L - \int_d^L \frac{hL}{(L-d)k\pi}\cos\left(\frac{k\pi x}{L}\right)dx$$

$$= \left[\frac{-h(L-x)L}{(L-d)k\pi}cos\left(\frac{k\pi x}{L}\right)\right]_d^L - \frac{hL}{(L-d)k\pi}\left[\frac{L}{k\pi}\sin\left(\frac{k\pi x}{L}\right)\right]_d^L$$

$$= \frac{-h(L-L)L}{(L-d)k\pi}cos\left(\frac{k\pi L}{L}\right) - \frac{h(L-d)L}{(L-d)k\pi}cos\left(\frac{k\pi d}{L}\right)$$

$$-\frac{hL}{(L-d)k\pi}\,\frac{L}{k\pi}\left(\sin\left(\frac{k\pi L}{L}\right) - \sin\left(\frac{k\pi d}{L}\right)\right)$$

$$= \frac{hL}{k\pi}cos\left(\frac{k\pi d}{L}\right) + \frac{hL^2}{(L-d)k^2\pi^2}\sin\left(\frac{k\pi d}{L}\right)$$

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Thus

$$A_k = \frac{2}{L}\left[\frac{-hL}{k\pi}cos\left(\frac{k\pi d}{L}\right)+\frac{hL^2}{k^2\pi^2 d}\sin\left(\frac{k\pi d}{L}\right)+\frac{hL}{k\pi}cos\left(\frac{k\pi d}{L}\right)+\frac{hL^2}{(L-d)k^2\pi^2}\sin\left(\frac{k\pi d}{L}\right)\right]$$

$$= \frac{2}{L}\left[\frac{hL^2}{k^2\pi^2 d}\sin\left(\frac{k\pi d}{L}\right)+\frac{hL^2}{(L-d)k^2\pi^2}\sin\left(\frac{k\pi d}{L}\right)\right]$$

$$= \frac{2}{L}\left(\frac{hL^2}{k^2\pi^2 d}\frac{\times(L-d)}{\times(L-d)}+\frac{hL^2}{(L-d)k^2\pi^2}\frac{\times d}{\times d}\right)\sin\left(\frac{k\pi d}{L}\right)$$

$$= \frac{2}{L}\left(\frac{hL^3-hL^2d+hL^2d}{(L-d)k^2\pi^2 d}\right)\sin\left(\frac{k\pi d}{L}\right)$$

$$= \frac{2hL^2}{(L-d)k^2\pi^2 d}\sin\left(\frac{k\pi d}{L}\right)$$

The solution:

$$A_k = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{k\pi x}{L}\right)dx = \frac{2hL^2}{\pi^2k^2d(L-d)}\sin\left(\frac{k\pi d}{L}\right)$$

The displacement function then becomes:

$$y(x,t) = \sum_{k=1}^{\infty}\frac{2hL^2}{\pi^2k^2d(L-d)}\sin\left(\frac{k\pi d}{L}\right)\sin\left(\frac{k\pi}{L}x\right)\cos\left(\frac{ck\pi}{L}t\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

3.4 Harmonics

Equation (8) states that the displacement function is a sum of terms called modes or harmonics. Each mode represents a harmonic motion with different wavelength. For a fixed time t_1:

$$y(x,t_1) = \sum_{k=1}^{\infty}\frac{2hL^2}{\pi^2k^2d(L-d)}\sin\left(\frac{k\pi d}{L}\right)\cos\left(\frac{ck\pi}{L}t_1\right) \times \sin\left(\frac{k\pi}{L}x\right)$$

Implying that the each mode is a constant times $\sin(k\pi x/L)$. As x runs from 0 to L, the argument of $\sin(k\pi x/L)$runs from 0 to kπ , which is k half- periods of sin.

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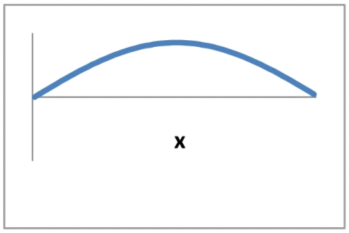

Fig: Amplitude for k=1

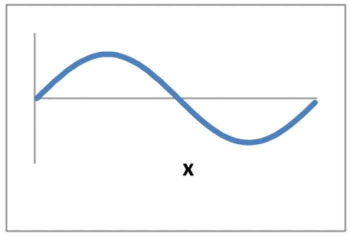

fig: Amplitude for k=2

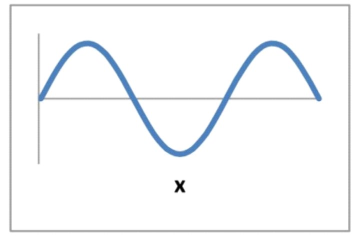

fig: Amplitude for k=3

Similarly, for any fixed position x1:

$$y(x_1, t) = \sum_{k=1}^{\infty} \frac{2hL^2}{\pi^2 k^2 d(L-d)} \sin\left(\frac{k\pi d}{L}\right) \sin\left(\frac{k\pi}{L}x_1\right) \quad \times \cos\left(\frac{ck\pi}{L}t\right)$$

Implying that each mode is a constant times $\cos(ck\pi t/L)$. As t increases from 0 to 1 s, the argument of $\cos(ck\pi t/L)$increases by $\frac{ck\pi}{L}$, which is $\frac{ck}{2L}$ cycles. For mode k=1 (the fundamental tone), the frequency is $\frac{c}{2L}$ cycles per second. For mode k = 2 (the second harmonic), the frequency is $2\frac{c}{2L}$ cycles per second. For mode k = 3 (the third harmonic), the frequency is $3\frac{c}{2L}$ cycles per second etc.

Since $c=\sqrt{T/\mu}$ the frequency of oscillation of a string decrease with the density and increase with the tension or by shortening the string.

3.5 Coefficients

The coefficients in (7) found by Fourier transformation gives the amplitude of each harmonic, i.e. computed by inserting k=1, k=2, k=3, etc. By superposition lemma the total displacement is formed by the sum of the harmonics.

To illustrate the principle, we consider the following example: L = 0.64, h = 0.005 and d = 0.16, giving the following amplitudes according to (7):

k	A_k
1	3.821E-03
2	1.351E-03
3	4.246E-04
4	4.138E-20
5	-1.528E-04

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We compute the graphs using Excel for t = 0 as an example. The first harmonic:

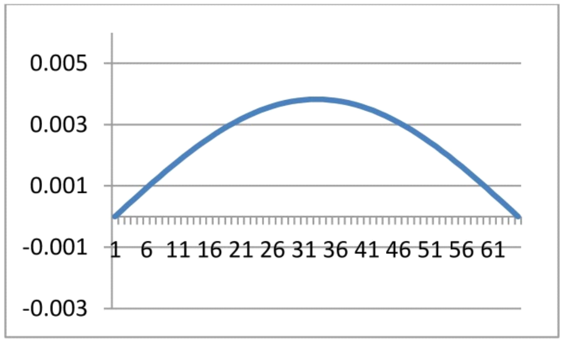
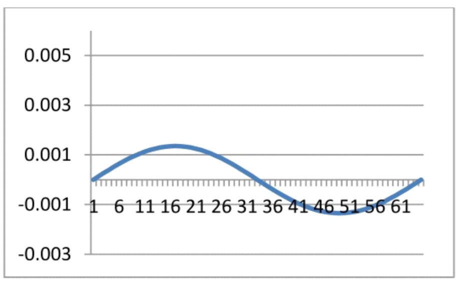


Figure: First harmonic at t=0.

The second harmonic and the sum of the first two harmonics:



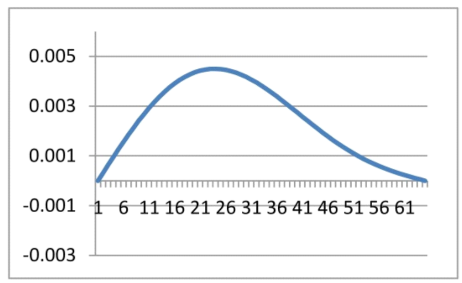
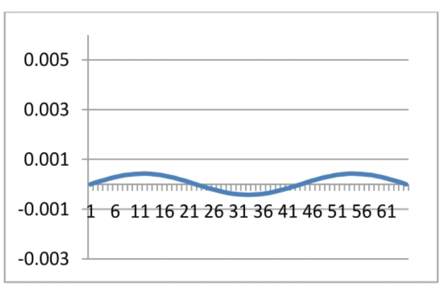


Fig: Second harmonic at t=0. **Fig: Sum of the two first harmonics at t=0.**

The third harmonic and the sum of the first three harmonics:



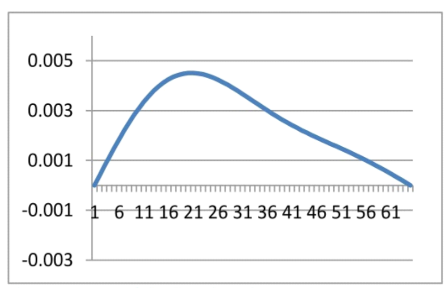


Figure: Third harmonic at t=0. **Figure: Sum of the first three harmonics at t=0**

As more harmonics are added, the total displacement function as the accumulated sum of harmonics becomes more and more similar to the initial displacement function f(x)

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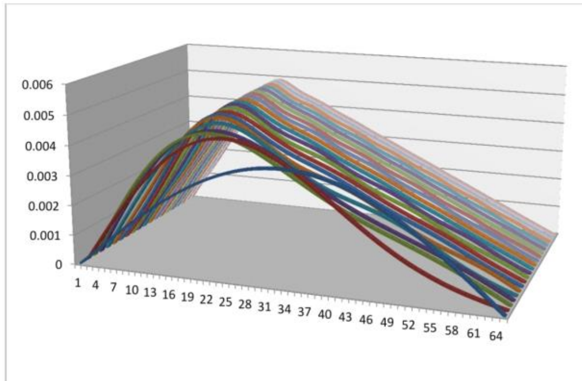

Figure: Accumulated sum of the first 20 harmonics for d = 0.16.

3.6 Amplitude distribution depending on position of plucking

To find the influence of the distribution of amplitudes for the harmonics based on the position of plucking, we compute amplitudes according to (7) for some example values of d. L and h are fixed (L = 0.64, h = 0.005).

d = 0.32

Plucking at the middle of the string (d = 0.32) should produce the following amplitude distribution:

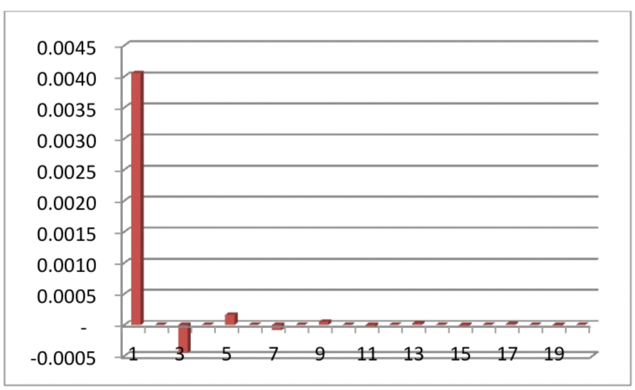

Figure: Amplitudes for d = 0.32.

Here the fundamental harmonic is very dominant, producing a “pure” sound with low amplitudes on overtones. Even-numbered harmonics (k = 2, k = 4, etc.) are completely missing.

d = 0.16

Plucking at the regular playing area of the Saroj Veena (cad = 0.16) produces a slightly different amplitude distribution.

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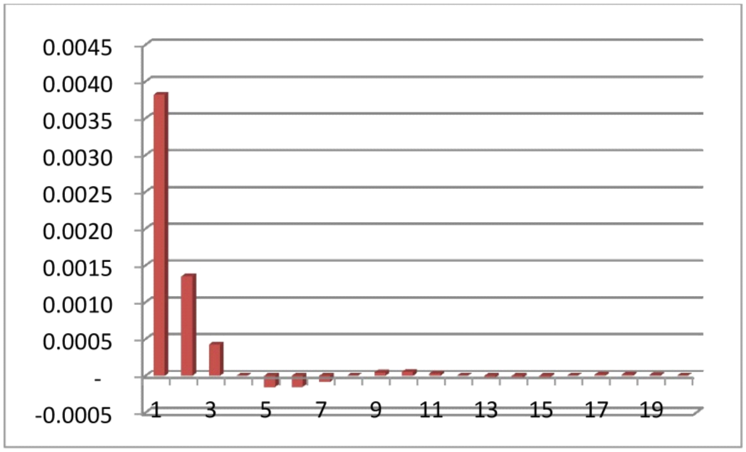

Figure: Amplitudes for d = 0.16.

The fundamental tone is still dominant, but the five first overtones would influence the tone quality.

d = 0.05

Plucking close to the nut or the bridge of the Saroj Veena should produce the following amplitude distribution.

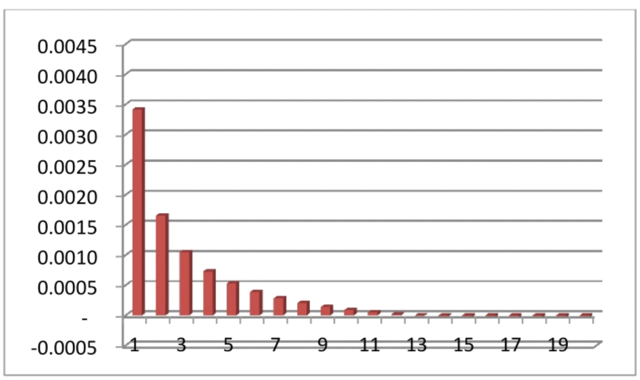

Figure: Amplitudes for d = 0.05.

In this case the overtones will be clearly more visible, even up to the 10^{th} overtone. This would create a ‘bright’ tone quality.

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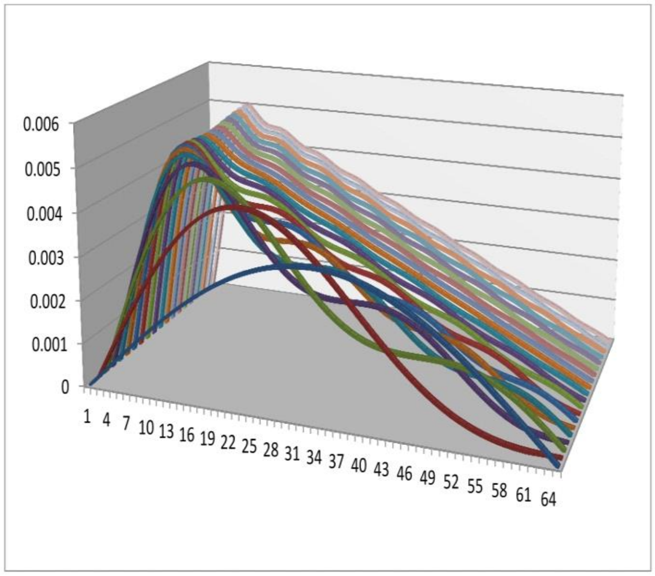

Figure: Accumulated sum of the first 20 harmonics for d = 0.05.

4. EXPERIMENT SETUP

A microphone was connected to a PC running the following software tools:

a. Audacity for recording from the microphone
b. Spectra Scope for analyzing the sound files and producing frequency amplitudes
c. MS Excel for examining and analyzing the output frequency tables

Using Audacity and recoding to a file enabled full control over the start and end times of the recording, avoiding transients at the beginning and distortion due to fading. Samples of 1.04 seconds were used.

Spectra Scope uses an implementation of a Fast Fourier Transform (FFT) to produce a frequency plot of the amplitudes given in dB. This allowed us to derive the relative energies in the fundamental tone and overtones. Spectra Scope supported export of frequency tables.

Excel was used to import and then analyze the frequency tables output form Spectra Scope. To reduce sources of error, dBv values from ca 10 Hz below to ca 10 Hz above the actual frequency were summarized. Each group always contained eight values. In order to get reproducible and consistent results, the position of plucking was accurately measured and tagged on the Saroj Veena. The sideways offset of 0.5 cm was also indicated.

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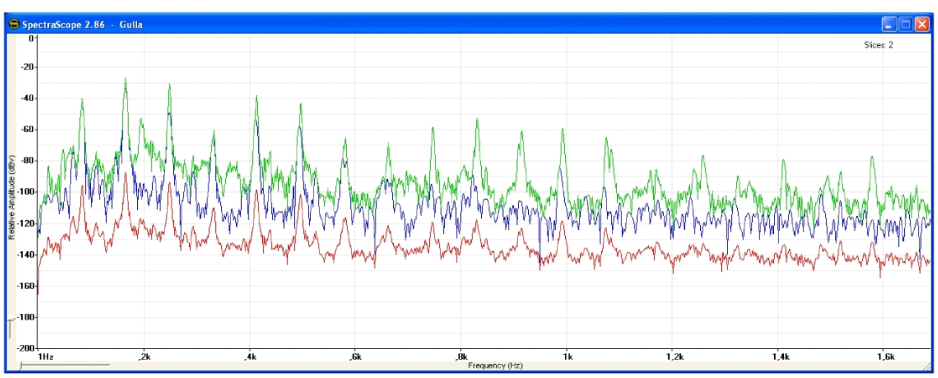
Saroj Veena

The origin of this instrument is Tripura, Gomati District. This Musical Instrument's playing technique has some similarities to Hindustani classical guitar (slide guitar). But the structure, shape, construction material and, above all, the sound production quality and tone are more similar to the Vichitra Veena. It is played using a left-hand slide and special plectrums on right-hand fingers. Outside a slide guitar's regular fretboard, everything in this instrument is indianised. The Instrument consists of one piece of solid wood. It has a semi flat chamber of sound, like the Tambura and Veena. It has four strings to play, three chikaris and 13 sympathetic strings to play. For improved sound sustainability and resonance a Chrome-plated Brass Tumba is screwed into the back of the neck. The edge of the hollow neck or fretboard is well curved to resemble a Peacock's neck. The Instrument's sound is more like the veena, very smooth and well balanced. Like Vichitra Veena, the beautiful sustained tone helps an artist play in the true (vocal) style of gayaki. The notes' position is marked with inlays to allow easy play. The Saroj Veena is under tremendous tension; it pulls the total strings to be over 500 pounds. The tone tunes incredible with the sympathetic ringing out and strengthening each note played is due to this high tension. This is a loud instrument made for small amplification to cut through. Now a day Saroj Veena is a Popular Musical Instrument of Tripura. This Instrument is played in various orchestras, Festival, Pujas. This Instrument is used in Folk Music; there is also the use of this instrument in Hindustani Classical Music.

4.1 Measurements

4.1.1 Saroj Veena (Acoustic), 82Hz, d=0.16, plucking with plastic plectrum

Using a high-quality microphone connected to a laptop, a recording of 1.04 second was done. Below the amplitude-to-frequency diagram of SpectraScope is shown.



The dBv values for different frequencies were imported into Excel for further processing.

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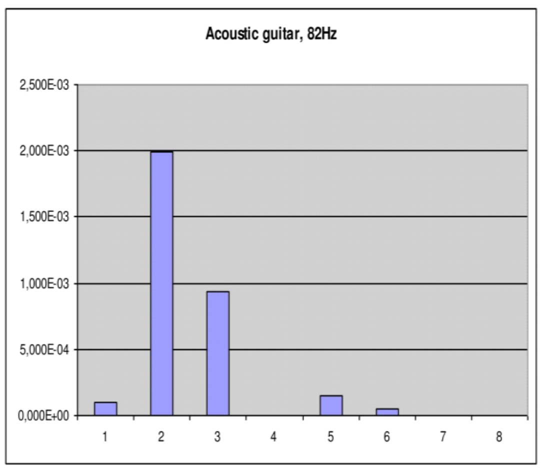
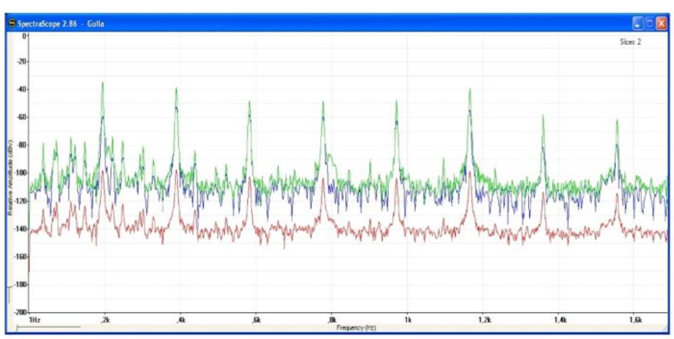
Tone	Ideal frequency (Hz)	Actual frequency (Hz) for max amplitude	Amplitude (dBv) from SpectraScope	Amplitude
Fundamental	82	82	-39.9	1.023E-04
Overtone 1	164	164	-27.0	1.995E-03
Overtone 2	246	247	-30.3	9.333E-04
Overtone 3	328	332	-60.3	9.333E-07
Overtone 4	410	413	-38.3	1.479E-04
Overtone 5	492	496	-43.2	4.786E-05
Overtone 6	574	581	-64.7	3.388E-07
Overtone 7	656	662	-68.2	1.514E-07

As observed from the diagram above, the amplitude for the fundamental tone is quite low for 82Hz. In an acoustic guitar the sound is mainly produced by the guitar body and not the string itself (J.Pelc n.d.). A hypothesis could be that the body was not able to reproduce such low tones.



4.1.2 Saroj Veena (Acoustic), 196 Hz, d=0.16, plucking with plastic plectrum

Instead of plucking the low E string, the experiment was redone with the G string at approximately 196 Hz.



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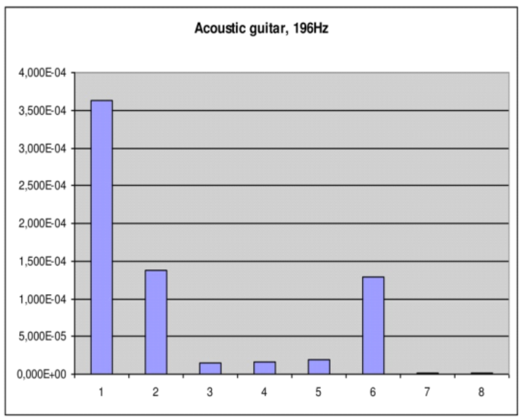
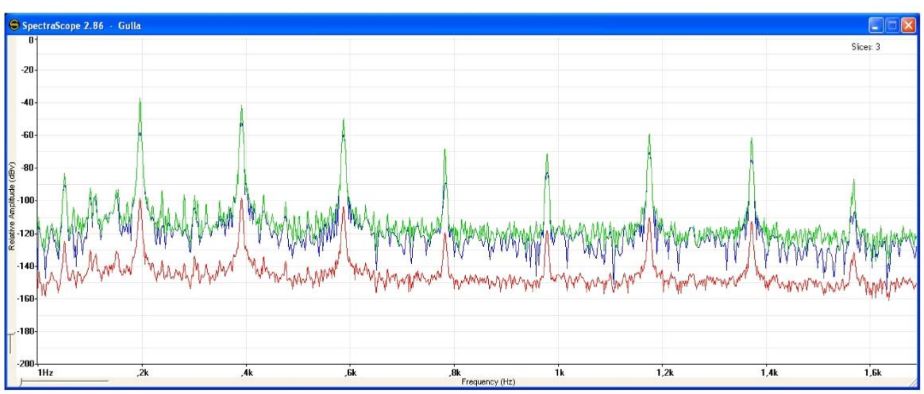
Tone	Ideal frequency (Hz)	Actual frequency (Hz) for max amplitude	Amplitude (dBv) from SpectraScope	Amplitude
Fundamental	196	193	-34.4	3.631E-04
Overtone 1	392	388	-38.6	1.380E-04
Overtone 2	588	582	-48.2	1.514E-05
Overtone 3	784	776	-48.0	1.585E-05
Overtone 4	980	971	-47.1	1.950E-05
Overtone 5	1176	1165	-38.9	1.288E-04
Overtone 6	1372	1359	-58.0	1.585E-06
Overtone 7	1568	1555	-61.1	7.762E-07



This graph matches the predicted distribution by the mathematical model much closer.

4.1.3 Electric Saroj Veena, 196Hz, d=0.16, plucking with plastic plectrum

To eliminate the resonance effect of the acoustic guitar body completely, an electric guitar was used. A digital external sound was used to amplify the signal produced by the electric guitar.



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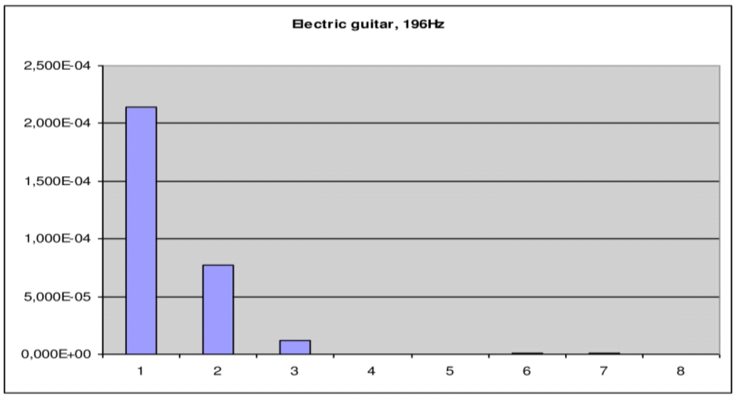
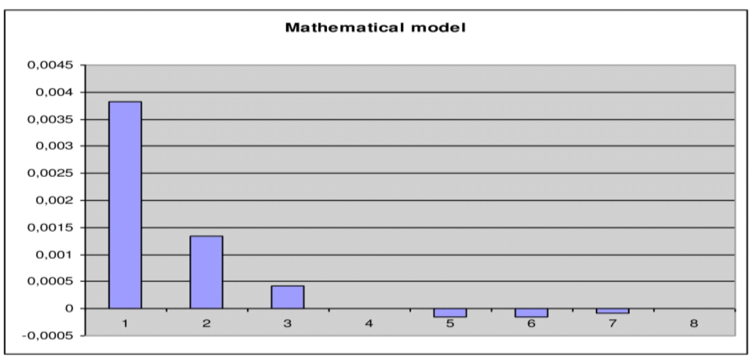
Tone	Ideal frequency (Hz)	Actual frequency (Hz) for max amplitude	Amplitude (dBv) from SpectraScope	Amplitude
Fundamental	196	195	-36.7	2.138E-04
Overtone 1	392	390	-41.1	7.762E-05
Overtone 2	588	586	-49.3	1.175E-05
Overtone 3	784	781	-68.2	1.514E-07
Overtone 4	980	978	-71.0	7.943E-08
Overtone 5	1176	1174	-59.1	1.230E-06
Overtone 6	1372	1372	-61.2	7.586E-07
Overtone 7	1568	1567	-86.8	2.089E-09



For comparison the corresponding amplitudes computed by formula (7) are shown below.



As can be observed the relative distribution of amplitudes for the different harmonics for an actual electrical guitar quite closely resemble the distribution predicted by the mathematical formula.

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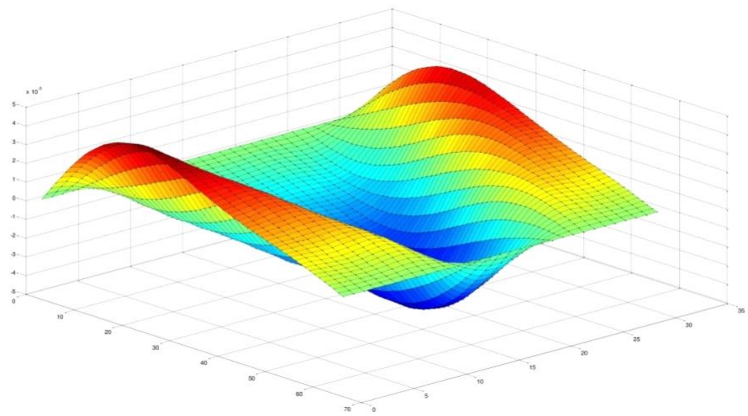

Figure: Displacement functions of time and position for the three first harmonics

5. CONCLUSION

The equilibrium displacement of each string segment (denoted by position x) at any time is described by equation (8). (Denoted by t). Because of the formula's simplicity, it's easier to think of the dynamic function as a superposition of modes or harmonics. Each harmonic defines a tone that corresponds to the potential standing waves given by the string length, with frequency dependent on string tension and density. The amplitudes of each harmonic are represented by the coefficients derived from the Fourier series (7). Based on this mathematical model, we may compute theoretical amplitudes for the plain tone and overtones, depending on the magnitude (h) and position (d) of the string plucking. When plucking towards the center of the string, the basic tone's amplitude is dominant, resulting in a clear, sine-like tone. More of the potential energy produced goes into higher-pitch harmonic tones when plucked closer to the edge of the line, creating a smoother note. This is also consistent with results using a genuine Saroj Veena.

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