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ANALYSIS OF FLOW CHARACTERISTICS OF THE BLOOD THROUGH CURVED ARTERY WITH MILD STENOSIS

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Abstract

Narrowing of the arteries caused by atherosclerosis reduces blood flow to the heart, which results shows ischemia, angina pectoris, cerebral strokes, and other coronary artery disease signs and symptoms. Curvature is seen in blood vessels at various locations. The stenotic surface provides an additional curvature and the point of maximum shear which varies with the cross-section. A cylindrical form of the Navier-Stokes equations in polar coordinate system have been extended to include dynamic curvature along the axial direction. The blood flow behavior of taking different values of blood parameters like viscosity, the radius of the artery, and the thickness of the stenosis has been studied with and without curvature by using an extended blood flow model with dynamic curvature. Moreover, the aspects of blood flow, such as dynamic curvature velocity profile, volumetric flow rate, pressure drop, and shear stress, have been studied in relation to blood flow around curved arteries with stenosis, variations in the radii of the artery, thickness of the stenosis, and viscosity. The information may reveal that by increasing the values of curvature, viscosity, and thickness of stenosis, velocity, and volumetric flow rate can be quickly reduced. Increasing the curvature, viscosity, and thickness of stenosis also results in an increase in shear stress and a pressure drop. The presence of curved stenotic arteries has a significant impact on the flow parameters, and it is crucial to know about these dynamics in order to study the cardiovascular system.

Keywords: Arterial Stenosis, Blood Viscosity, Curvature, Velocity Profile, Volumetric Flow Rate, Pressure Drop, Shear Stress.

1. INTRODUCTION

Deposition of cholesterol, fatty particles, and other foreign particles form atherosclerotic plaque, abnormal tissue development causes stenosis [2, 29]. Unusual hemodynamic conditions lead to an aberrant biological reaction, and blood vessel flow is closely linked to cardiovascular disorders [8, 24]. Any kind of fat buildup in the artery wall obstructs blood flow and reduces the accessible amount of oxygen [18]. Stenosis in an artery generally occurs due to the aggregation of cholesterol-laden plaque in its walls resulting in a constricting of the passage of blood as well as a loss of elasticity that leads to stroke and heart attack [21]. Stroke is one of the important causes of death that follows heart disease, the cause of stroke is the blockage of blood vessel [16]. The main factors affecting blood flow in the stenosed portion of the artery include the deformability of

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erythrocytes, wall shear stress, and viscosity of the blood [14]. There are various types of stenosis, and the role of each of these causes obstruction in the blood flow due to the deposition of foreign bodies [7, 25]. The major effect of stenosis are angina pectoris, heart attacks, ischemia, and cerebral strokes [12]. The fundamental aspect of many diseases, including their diagnosis and treatment, requires an understanding of fluid dynamical features such as velocity profile, velocity gradient, shear stress, the pressure drop of blood, the deformability of red cells, and boundary conditions [5, 6]. Flow disturbances are enhanced and established once plaque develops and encroaches the lumen [10]. The dual nature of blood rheology, where it can exhibit Newtonian behavior under certain conditions (high shear rates, large artery diameters) but behaves as a non-Newtonian fluid under other conditions (low shear rates, small artery radii) [15, 18, 22]. Researchers have theoretically and experimentally investigated the impact of constriction on blood flow characteristics like flow parameters, viscosity, and resistance [7, 9, 12, 21]. The interdisciplinary nature of the research, involving collaboration between mathematicians and medical researchers. It also emphasizes the practical application of the research through the development and empirical verification of an equation for projecting hemodynamic factors in stenotic regions, which is relevant to understanding and managing vascular conditions [3, 7, 27, 30].

Curvature is seen in blood vessels at various locations. Blood flow encounters bending along its route, causing changes in velocity, pressure drop, volumetric flow rate, and shear stress that affects the motion of the flow [14]. Local variations in curvatures can indeed have a significant impact on the flow parameters, influencing factors like centripetal forces, and acceleration along a path [28]. Deformation parameter of the curvature is a ratio of the change in curvature to the mean radius of curvature [1]. Curvature effects not only in the blood flow but also in oxygen transport, different types of flow are possible in the curved part of the artery such as laminar, steady, and phasic [14, 29]. Such aggregates are ruptured and blood starts to flow with a very small finite stress. In both steady and phasic flow models of a curved coronary artery with different levels of stenosis, Computational fluid dynamics techniques are used to calculate the velocity and wall shear stress distributions [29]. Stenosis affects arterial hemodynamics by altering wall shear stress, increasing flow resistance, and influencing the non-Newtonian behavior of blood [13]. Despite the fact that the exact mechanisms prompting the formation of curved arterial stenosis are not yet understood but remedies for a such cardiovascular disorder is imperative to save countless life [7, 22].

Singh et al. [2] have assumed a mild stenosis which is radially non-symmetrical. Chakravarty [27] observed stenotic deposits are in irregular shapes, cosine-type or smooth regular shapes are generally found to be considered in three-dimensional modeling of blood flow in arteries. Chakaravarty [27] through analytical research in which he regarded blood as a non-Newtonian fluid that followed the power-law model, and demonstrated how mild stenosis in smaller arteries affects the way the blood flows in terms of resistance. The same geometry of the stenosis mentioned in all studies on stenotic flow is pointed out by Young [7]. Santamarina et al. [1] Studied the complexities of blood flow through curved vessels with dynamic changes in curvature, and the findings

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contributed knowledge of cardiovascular physiology and pathology for the diagnosis and treatment of conditions related to coronary artery flow. Dash et al. [23] derived an appropriate analytic solution to the problem of blood flow through a catheterized artery with a small curvature and mild stenosis, and investigated stenosis's effects on the flow be more important than the curvature. Padmanabhan and Jayaraman [19] developed an analytical solution to a mathematical modeled blood flow problem in a curving stenosed artery using toroidal coordinate and perturbation theory. Schilt et al. [28] have studied an in vitro flow model containing a flexible, curved tube through which fluid moves while being subjected to a steady state pressure gradient. Liu [4] the effect of stenosis on the pulsatile blood flow pattern in curved arteries with stenosis at the inner wall was studied using computer models, when compared to a curved artery without stenosis, the flow pattern downstream of the artery exhibited a significant difference. Lori et al. [11] studied the impact of curvature on the transport system within arterio-venous fistulae and demonstrated that curvature significantly affects the flow pattern of oxygen and blood transport. Chakravarty and Chowdhury [26] performed a quantitative examination of the blood flow behavior in a stenosed curved artery and explored how the stenosis form affects flow resistance. Alastruey et al. [17] analyzed the relationship between vascular curvature and blood flow by using the image data to examine steady-state flow in a range of vascular variations, such as single and double bends, suggests a comprehensive approach to understanding the biomechanics of blood circulation. Qiu and Trabell [31] studied the intricate relationships between blood flow dynamics, oxygen transport, and vascular biomechanics in the conditions of a curved coronary artery.

This article examines blood flow parameters in curved arteries by incorporating the curvature factor into the axial Navier-Stokes equations. It analyzes fluid dynamics, including velocity profiles, volumetric flow rate, pressure drops, and shear stress, under conditions of stenosis, by solving extended model for blood flow through curved artery with specific boundary conditions.

2. EXTENDED MODEL FOR BLOOD FLOW THROUGH CURVED ARTERY

Flow inside a cylinder, blood flow in arteries can be modeled. Let *r* and *p* stand for the artery's blood flow's radius and pressure drop respectively. Components of the velocities along radial, angular and axial directions are v^r, v^θ and v^z respectively. The system can be expressed in cylindrical polar coordinate form (r,θ,z). The equation of continuity and the equation of motion with the curvature effect in the axial direction are [15, 18]

$$\frac{\partial v^r}{\partial r}+\frac{v^r}{r}+\frac{1}{r}\frac{\partial v^\theta}{\partial \theta}+\frac{\partial v^z}{\partial z}=0, \tag{1}$$

$$\rho\left(\frac{\partial v^r}{\partial t}+v^r\frac{\partial v^r}{\partial r}+\frac{v^\theta}{r}\frac{\partial v^r}{\partial \theta}-\frac{(v^\theta)^2}{r}+v^z\frac{\partial v^r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}(rv^r)\right)+\frac{1}{r^2}\frac{\partial^2 v^r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v^\theta}{\partial \theta}+\frac{\partial^2 v^r}{\partial z^2}\right] \tag{2}$$

$$\partial v^\theta \quad {}_r\,\partial v^\theta \quad v^\theta\,\partial v^\theta \quad v^r v^\theta \quad \partial v^\theta \quad 1\,\partial p \quad 1\;\partial \quad \partial \quad {}_\theta \quad 1\;\partial^2 v^\theta$$

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$$\rho\left(\frac{}{\partial t}+v\ \frac{}{\partial r}+\frac{}{r}\frac{}{\partial\theta}+\frac{}{r}+v^z\frac{}{\partial z}\right)=-\frac{}{r}\frac{}{\partial\theta}+\mu\left[\frac{}{r}\frac{}{\partial r}\left(\frac{}{\partial r}(rv\ )\right)+\frac{}{r^2}\frac{}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial v^r}{\partial\theta}+\frac{\partial^2v^\theta}{\partial z^2}\right] \quad (3)$$

$$\rho\left(\frac{\partial v^z}{\partial t}+v^r\frac{\partial v^z}{\partial r}+\frac{v^\theta}{r}\frac{\partial v^z}{\partial\theta}+v^z\frac{\partial v^z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v^z}{\partial r}+\frac{\kappa P}{16\mu}(R^2-r^2)^2\right)+\frac{1}{r^2}\frac{\partial^2v^z}{\partial\theta^2}+\frac{\partial^2v^z}{\partial z^2}\right] \quad (4)$$

where κ is the dynamic curvature along the axial direction. We assume that in the axisymmetric shape, $v^\theta = 0$ and that v^r, v^z, and p are independent of θ. For the steady flow of blood, viscosity μ and density ρ are considered to be constant. Then, mass balance and momentum balance equation (1)-(4) reduced to

$$\frac{1}{r}\frac{\partial}{\partial r}(rv^r)+\frac{\partial}{\partial z}(v^z) = 0 \quad (5)$$

$$\rho\left(\frac{\partial v^r}{\partial t}+v^r\frac{\partial v^r}{\partial r}+v^z\frac{\partial v^r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left(\frac{\partial^2v^r}{\partial r^2}+\frac{\partial^2v^r}{\partial z^2}+\frac{1}{r}\frac{\partial v^r}{\partial r}-\frac{v^r}{r^2}\right) \quad (6)$$

$$\rho\left(\frac{\partial v^z}{\partial t}+v^r\frac{\partial v^z}{\partial r}+v^z\frac{\partial v^z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{\partial^2v^z}{\partial r^2}+\frac{\partial^2v^z}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(v^z+\frac{\kappa P}{16\mu}(R^2-r^2)^2\right)\right] \quad (7)$$

We have considered axially symmetrically flow along z-axis only, so $v^r = 0$, $v^\theta = 0$, and $v^z = v(r)$ which is velocity component parallel to z-axis, then equations (5)-(7) become

$$\frac{\partial v}{\partial z} = 0, \quad 0=-\frac{\partial p}{\partial r}, \quad 0=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{\kappa P}{4\mu}(R^2-r^2)\right) \quad (8)$$

Suppose pressure term as $P = -\partial p/\partial z$, equation (8) reduces to

$$-P\frac{r}{\mu}=\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}+\frac{\kappa P}{16\mu}\ (R^2-r^2))^2\right). \quad (9)$$

where $P=-\frac{\partial p}{\partial z}$. The boundary condition is

$$v=\begin{cases}0 & \text{at} \quad r=R\\ 0 & \text{at} \quad r=R_0\end{cases} \quad (10)$$

and $\frac{\partial v_z}{\partial r}=0$ at $r=0$.

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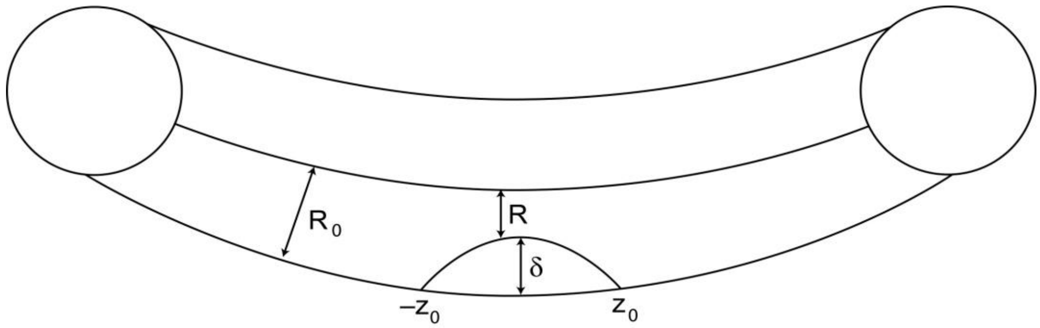

A

Figure 1: A: Sketch of blood flow in curved stenosed artery

2.1 Geometry of Stenosis

The geometry of stenosis developed is considered abnormal tissue buildup in the inner wall of a curved artery is shows in figure 1 where the ratio of the radii with and without stenosis is modeled as [15]

$$\frac{R}{R_0} = \begin{cases} 1 - \frac{\delta}{2R_0}\left(1 + cos\frac{\pi z}{z_0}\right) & \text{for} \quad |z| \leq z_0 \\ 1 & \text{for} \quad |z| > z_0 \end{cases} \tag{11}$$

where R is artery radius with stenosis, R_0 is the artery radius without stenosis and δ, is maximum height of stenosis.

2.2 Velocity of Blood Flow in Curvature Model

Integrating curvature model (9) ,

$$-P\frac{r^2}{2\mu} + A(z) = r\frac{\partial v_c}{\partial r} + \frac{\kappa P}{16\mu} \quad R^2 - r^2)^2 \tag{12}$$

Using boundary condition, $\frac{\partial v_c}{\partial r} = 0$ at r= 0, then equation (12) yields

$$A(z) = \frac{\kappa P R^4}{16\mu}$$

Then,

$$\frac{\partial v_c}{\partial r} = -\frac{Pr}{2\mu} + \frac{\kappa P R^2 r}{8\mu} - \frac{\kappa P r^3}{16\mu}$$

On integration

$$v_c = -\frac{Pr^2}{4\mu} + \frac{\kappa P R^2 r^2}{16\mu} - \frac{\kappa P r^4}{64\mu} + B(z)$$

Using $v_c = 0$ at $r = R$ gives

$$B(z) = \frac{PR^2}{4\mu} - \frac{3PR^4}{64\mu}$$

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After substituting value of *B*(*z*)

$$v_c = \frac{P}{4\mu}\ \ R^2 - r^2) + \frac{\kappa P}{64\mu}\ \ 4R^2r^2 - 3R^4 - r^4). \qquad (13)$$

This is the velocity distribution function for blood flow in curved artery.

The velocity distribution without curvature (i.e $\kappa = 0$) is

$$v_0 = \frac{P}{4\mu}\ \ R^2 - r^2) \qquad (14)$$

2.3 Volumetric Flow Rate in Curved Artery

The curved artery's volumetric flow rate is

$$Q_c = \int_0^R 2\pi r dr v_c = \int_0^R 2\pi r \left[\frac{P}{4\mu}\ \ R^2 - r^2) + \frac{\kappa P}{64\mu}\ \ 4R^2r^2 - 3R^4 - r^4)\right] dr$$

After integration and simplification

$$Q_c = \frac{\pi P R^4}{8\mu}\left(1 - \frac{\kappa R^2}{6}\right)$$

Using the geometry

$$Q_c = \frac{\pi P\left(R_0 - \frac{\delta}{2}\left(1 + cos\frac{\pi z}{z_0}\right)\right)^4}{8\mu}\left(1 - \frac{\kappa}{6}\left(R_0 - \frac{\delta}{2}\left(1 + cos\frac{\pi z}{z_0}\right)\right)^2\right)$$

Applying the binomial expansion, moreover the maximum height of stenosis is at $z = 0$.

$$Q_c = \frac{\pi P R_0^4}{8\mu}\left(1 - \frac{4\delta}{R_0}\right)\left(1 - \frac{\kappa R_0^2}{6}\left(1 - \frac{2\delta}{R_0}\right)\right)$$

$$Q_c = \frac{\pi P R_0^4}{8\mu}\left(1 - \frac{4\delta}{R_0} - \frac{\kappa R_0^2}{6}\left(1 - \frac{6\delta}{R_0} + \frac{8\delta^2}{R_0^2}\right)\right) \qquad (15)$$

In particular, the volumetric flow rate through stenosed artery without curvature effect (i.e., $\kappa = 0$) is

$$Q_0 = \frac{\pi P R_0^4}{8\mu}\left(1 - \frac{4\delta}{R_0}\right) \qquad (16)$$

2.4 Pressure Drop across the Stenosis with Curvature

Pressure drop across stenosis is

$$\Delta P = \int_{-z_0}^{z_0} P dz \qquad (17)$$

with the help of equation 15

$$\Delta P = \int_{-z_0}^{z_0} \frac{8\mu Q_c}{\pi R_0^4\left(1 - \frac{4\delta}{R_0}\right)\left(1 - \frac{\kappa R_0^2}{6}\left(1 - \frac{2\delta}{R_0}\right)\right)} dz$$

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Again using the binomial expansion, neglect higher power of δ, and then integration,

$$\Delta P = \frac{16\mu Q_c z_0}{\pi R_0^4}\left(1+\frac{4\delta}{R_0}+\frac{\kappa R_0^2}{6}\left(1+\frac{2\delta}{R_0}-\frac{8\delta^2}{R_0^2}\right)\right) \qquad (18)$$

The velocity distribution without curvature (i.e κ = 0) is

$$(\Delta P)_0 = \frac{16\mu Q_c z_0}{\pi R_0^4}\left(1+\frac{4\delta}{R_0}\right) \qquad (19)$$

2.5 Shear Stress across the Stenosis in Curved Artery

The stress across the stenosis in the curved artery is

$$\tau_s^c = \left[-\mu\frac{\partial v_c}{\partial r}\right]_{r=R} = \left[(-\mu)(-P)\frac{r}{2\mu}\right]_{r=R} = \frac{PR}{2}$$

Using equation (15)

$$\tau = \frac{8\mu Q_c}{\pi R_0^4\left(1-\frac{4\delta}{R_0}\right)\left(1-\frac{\kappa R_0^2}{6}\left(1-\frac{2\delta}{R_0}\right)\right)}\frac{R}{2}$$

$$= \frac{4\mu Q_c}{\pi R_0^4\left(1-\frac{4\delta}{R_0}\right)}\left(1-\frac{\kappa R_0^2}{6}\left(1-\frac{2\delta}{R_0}\right)\right)^{-1} R_0\left(1-\frac{\delta}{R_0}\right)$$

Again using the binomial expansion, neglect higher power of δ, and after simplification,

$$\tau = \frac{4\mu Q_c\left(1-\frac{\delta}{R_0}+\frac{\kappa R_0^2}{6}\left(1-\frac{3\delta}{R_0}+\frac{2\delta^2}{R_0^2}\right)\right)}{\pi R_0^3\left(1-\frac{4\delta}{R_0}\right)} \qquad (20)$$

In particular, the shear rate through stenosed artery without curvature effect (i.e., κ = 0) is

$$(\tau)_0 = \frac{4\mu Q_c\left(1-\frac{\delta}{R_0}\right)}{\pi R_0^3\left(1-\frac{4\delta}{R_0}\right)} \qquad (21)$$

3. RESULTS AND DISCUSSIONS

This section covers the velocity profile, volumetric flow rate, pressure drop, and shear stress of blood flowing through a curved artery with mild stenosis.

3.1 Velocity Profile by Variation of Curvature and Viscosity

Figure 2A describes the relation between velocity and radius for different values of curvature for constant viscosity (μ= 2.65 gram $mm^{-1}s^{-1}$). For each value of curvature, velocity is maximum at the center and decreases gradually towards wall. The two values, pressure 100 mm of Hg and radius of artery 1 mm are kept constant. When the curvature value is 0.00 mm^{-1} s, the maximum velocity at the center is 9.433 mm s^{-1}, and 6.096 mm s^{-1}, when the curvature is 2.0 mm^{-1} s.

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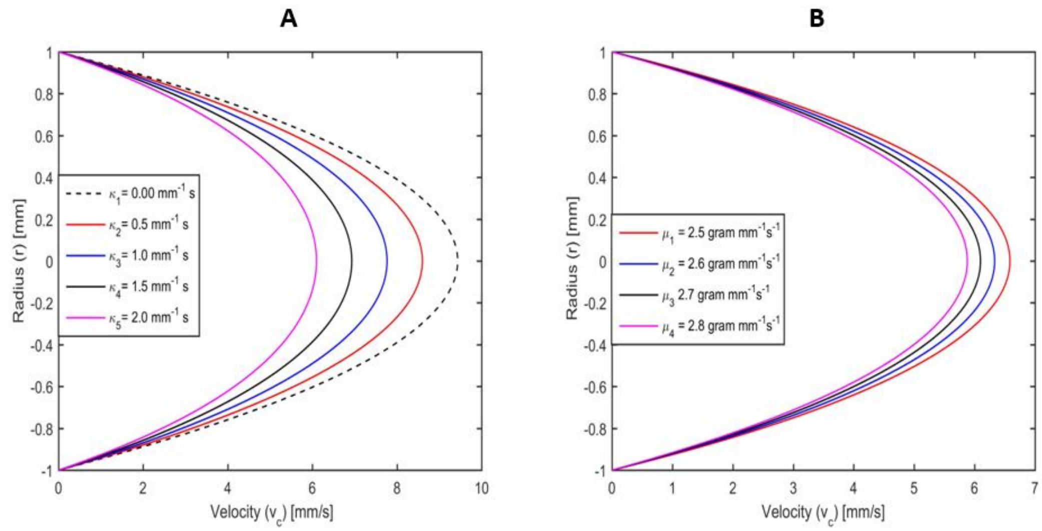

Figure 2: Velocity profiles A: at different radii and curvature B: at different radii and viscosity

Therefore, effect of curvature on velocity is seen in this figure. When the curvature of artery increases from 0.00 mm^{-1} s to and 0.5 mm^{-1} s, velocity of blood at the center of artery decreases from 9.433 mm s^{-1} to 8.592 mm s^{-1} which is about 0.841 mm s^{-1}, similarly when the curvature increases from 0.5 mm^{-1} s to 1.0 mm^{-1} s, the velocity decreases from 8.592 mm s^{-1} to 7.765 mm s^{-1} which is about 0.827 mm s^{-1} again. When the curvature incerases from 1.0 mm^{-1} s to 1.5 mm^{-1} s, velocity decreases from 7.765 mm s^{-1} to 6.93 mm s^{-1} which is about 0.835 mm^{-1} s at r = 0 mm. Finally, when the curvature increases from 1.5 mm^{-1} s to 2.0 mm^{-1} s, the velocity decreases from 6.93 mm^{-1} s to 6.096 which is about 0.834 mm^{-1} s. The conclusion from this analysis is that the velocity at the center of the curved artery is decreasing uniformly with constant amount for increasing curvature within this range.

Figure 2B shows the relationship between the radius of the artery and velocity for different values of viscosity. Velocity decreases for increasing viscosity. At the beginning, when viscosity is 2.5 gram $mm^{-1}s^{-1}$, and radius is 0 mm the velocity is 6.579 mm s^{-1}. Similarly, 6.331 mm s^{-1}, 6.097 mm s^{-1}, and 5.879 mm s^{-1} for the viscosity 2.6 gram $mm^{-1}s^{-1}$,2.7 gram $mm^{-1}s^{-1}$, and 2.0 gram $mm^{-1}s^{-1}$ respectively. When the radius of the artery increases 0.5 mm, the velocity are 4.995 mm s^{-1}, 4.803 mm s^{-1}, 4.625 mm s^{-1}, and 4.460 mm s^{-1} for the viscosity 2.5 gram $mm^{-1}s^{-1}$, 2.6 gram $mm^{-1}s^{-1}$, 2.7 gram $mm^{-1}s^{-1}$ and 2.8 gram $mm^{-1}s^{-1}$ respectively. Similarly, for the radius of the artery 1.0 mm the velocity are 0 for all values of viscosity. From this we conclude that the velocity decreases for increasing viscosity, when the radius of artery is kept constant. Similarly, velocity also decreases with increases radius of the artery for constant viscosity.

3.2 Volumetric Flow Rate for Variation of Curvature and Viscosity

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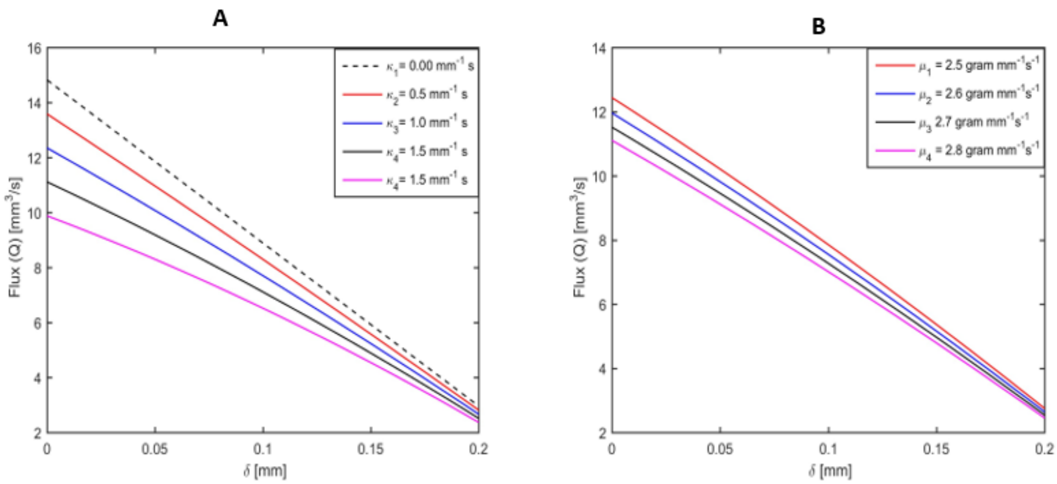

Figure 3: Volumetric flow rate A: at various heights of stenosis and curvature B: at different heights of stenosis and viscosity

Figure 3A demonstrates the relationship between volumetric flow rate and thickness of stenosis for different values of curvature for constant viscosity (μ = 2.65 gram mm^{-1}s^{-1}). For each value of curvature, volumetric flow rate is maximum at δ = 0 and decreasing linearly. The two values, pressure 100 mm of Hg and radius of artery 1 mm are kept constant. When the curvature value is 0.00 mm^{-1} s, the approximate maximum volumetric flow rate at δ = 0 is 14.82 mm^3 s^{-1} and 9.879 mm^3 s^{-1} only when the curvature is 2.0 mm^{-1}s. Therefore effect of curvature on volumetric flow rate is seen in this figure. When the curvature of artery increases from 0.00 mm^{-1} s to and 0.5 mm^{-1} s, volumetric flow rate at δ = 0 decreases from 14.82 mm^3 s^{-1} to 13.58 mm^3 s^{-1} which is about 1.24 mm^3 s^{-1}. Similarly when the curvature increases from 1.0 mm^{-1} s to 1.5 mm^{-1} s, the volumetric flow rate decreases from 12.35 mm^3 s^{-1} to 11.11 mm^3 s^{-1}, which is about 1.24 mm^3 s^{-1} again. Finally when the curvature increases from 1.5 mm^{-1} s to 2.0 mm^{-1} s, volumetric flow rate decreases from 11.11 mm^3 s^{-1} to 9.879 mm^3 s^{-1} which is about 1.231 mm^3 s^{-1}.

Table 1: Volumetric flow rate with curvature and viscosity for different height of stenosis

δ (mm)	**0.00**	**0.05**	**0.1**	**0.15**	**0.2**	%(Q)	κ
Q	14.82	11.82	8.867	5.916	2.964	20	0.00
Q	13.58	10.93	8.277	5.571	2.816	20.73	0.5
Q	12.35	10.05	7.686	5.226	2.667	21.51	1.0
Q	11.11	9.164	7.096	4.881	2.519	22.67	1.5
Q	9.879	8.278	6.505	4.536	2.371	24	2.0
δ (mm)	**0.00**	**0.05**	**0.1**	**0.15**	**0.2**	%(Q)	μ
Q	12.44	10.18	7.835	5.357	2.749	22.09	2.5
Q	11.96	9.791	7.533	5.151	2.643	22.09	2.6
Q	11.51	9.428	7.254	4.96	2.545	22.11	2.7
Q	11.10	9.092	6.995	4.783	2.454	22.11	2.8

This figure shows that as the height of stenosis increases the volumetric flow rate is decreases and its give minimum value at δ = 0.2 mm. The conclusion from this analysis

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is that the volumetric flow rate of the curved artery is decreasing uniformly with constant amount for increasing curvature and height of stenosis within this range. Figure 3B shows the relationship between the thickness of the stenosis and volumetric flow rate for different values of viscosity. Volumetric flow rate decreases for increasing viscosity. At the beginning, when the thickness of the stenosis is 0, and viscosity is 2.5 gram $mm^{-1}s^{-1}$, the volumetric flow rate is 12.44 $mm^3\ s^{-1}$, similarly 11.96 $mm^3\ s^{-1}$, 11.51 $mm^3\ s^{-1}$, and 11.10 $mm^3\ s^{-1}$ for the viscosity 2.6 gram $mm^{-1}s^{-1}$, 2.7 gram $mm^{-1}s^{-1}$, and 2.8 gram $mm^{-1}s^{-1}$ respectively. For the stenosis thickness 0.2 mm the volumetric flow rate are 2.749 $mm^3\ s^{-1}$, 2.643 $mm^3\ s^{-1}$, 2.545 $mm^3\ s^{-1}$, and 2.454 $mm^3\ s^{-1}$ for 2.5 gram $mm^{-1}s^{-1}$, 2.6 gram $mm^{-1}s^{-1}$, 2.7 gram $mm^{-1}s^{-1}$, and 2.8 gram $mm^{-1}s^{-1}$ respectively. From these results we conclude that the volumetric flow rate decreases for increasing viscosity, when the stenosis height is kept constant and volumetric flow rate also decreases with height of the stenosis for constant curvature. Table 1 describes the relationship between increasing thickness of stenosis and volumetric flow rate for different values of curvature. Height of the stenosis increases gradually from 0.00 to 0.2 mm and viscosity is kept constant. For each value of curvature volumetric flow rate at 0.00 and at 0.2 mm are compared. When the curvature is 0.00 mm^{-1} s, the volumetric flow rate decreases by 20%.

As the curvature increases the volumetric flow rate percentage increases gradually. Which is shown in the table. The decrement percentage increases gradually by 3.76%, 3.8%, 5.4%, and 5.9% for the curvature 0.00 mm^{-1} s to 0.5 mm^{-1} s, 0.5 mm^{-1} s to 1.0 mm^{-1} s, 1.0 to 1.5 mm^{-1} s, and 1.5 to 2.0 mm^{-1} s respectively. When the curvature increases from 0.00 mm^{-1} s to 2.0 mm^{-1} s, the volumetric flow rate decreases by 33.34%, but in case of viscosity the volumetric flow rate is decreased by 10.78%, at $\delta = 0$. In this case effect of curvature is more than the viscosity.

Similarly, at the lower part of the table shows relationship between increasing stenosis and volumetric flow rate for different values of viscosity. In this case curvature to keep constant. For each value off viscosity volumetric flow rate at 0.00 and 0.2 mm are compared. When the viscosity is 2.5 gram $mm^{-1}s^{-1}$ the percentage decrement in volumetric flow rate is 22.09%. The percentage of increment of volumetric flow rate are 22.09 and 22.11 for the viscosity 2.5 gram $mm^{-1}s^{-1}$, 2.6 gram $mm^{-1}s^{-1}$ and 2.7 gram $mm^{-1}s^{-1}$ gram $mm^{-1}s^{-1}$, 2.8 gram $mm^{-1}s^{-1}$ respectively. This shows that the percentage of increment of volumetric flow rate are almost uniform.

3.3 Comparison of Pressure Drop by Variation of Curvature and Viscosity

Figure 4A describes the relationship between pressure drop and thickness of stenosis for different values of curvature and constant viscosity (μ = 2.65 gram $mm^{-1}s^{-1}$). For each value of curvature, pressure drop is minimum at δ = 0.0 mm and increases linearly. The two values, pressure 100 mm of Hg and radius of artery 1.0 mm are kept constant. When the curvature value is 0.00 mm^{-1} s, the approximate minimum pressure drop at $\delta = 0$ is 50.61 mm of Hg and 67.48 mm of Hg only when the curvature is 2.0 mm^{-1} s.

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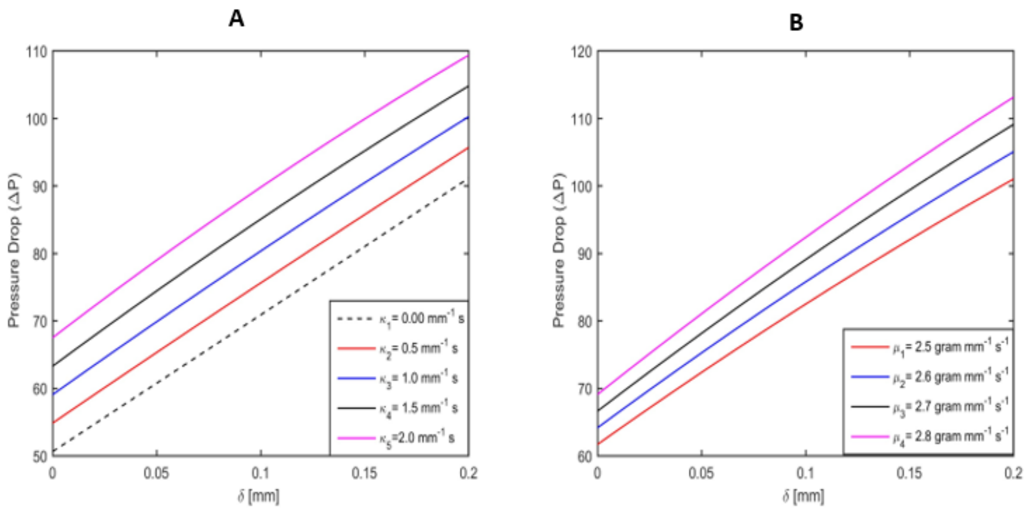

Figure 4: Pressure drops A: in the case of various heights of stenosis and curvature B: at various heights of stenosis and different viscosity

Therefore the effect of curvature on pressure drop is seen in this figure. when the curvature increases from 0.5 mm^{-1} s to 1.0 mm^{-1} s, the pressure drop increases from 54.83 mm of Hg to 59.05 mm of Hg which is about 4.22 mm^{-1} s again. When the curvature increases from 1.0 mm^{-1} s to 1.5 mm^{-1} s, the pressure drop increases from 59.05 mm of Hg to 63.26 mm of Hg which is about 4.22 mm of Hg. Finally, when the curvature incrases from 1.5 mm^{-1} s to 2.0 mm^{-1} s, the pressure drop increases from 63.26 mm of Hg to 67.48 mm of Hg which is about 4.22 mm of Hg. This shows that when the curvature increases uniformly the pressure drop also increases uniformly. The conclusion from this analysis is that the pressure drop at $\delta = 0$ of the curved artery is increasing uniformly with constant amount for increasing curvature and increases pressure drop with increasing the height of stenosis within this range. Figure 4B depicts the pressure drop in curved arteries with different heights of stenosis and for different values of viscosity μ. Viscosity μ takes (2.5,2.6,2.7,2.8) gram mm^{-1} s^{-1}, the pressure drops at $\delta = 0$ are 61.67 mm of Hg, 64.14 mm of Hg, 66.61 mm of Hg, and 69.07 mm of Hg respectively. For the stenosis thickness 0.1 mm, the pressure drops are 82.52 mm of Hg, 85.82 mm of Hg, 89.12 mm of Hg, and 92.42 mm of Hg for the viscosity (2.5,2.6,2.7,2.8) gram mm^{-1} s^{-1} respectively. Finally, when the stenosis thickness is 0.2 mm, the pressure drops are 101 mm of Hg, 105 mm of Hg, 109.10 mm of Hg, and 113.10 mm of Hg for the viscosity (2.5,2.6,2.7,2.8) gram mm^{-1} s^{-1}. From these results we conclude that the pressure drop increases for increasing viscosity, when the stenosis height is kept constant. Similarly pressure drop also increases with height of the stenosis for constant curvature.

3.4 Shear Stress by Variation of Curvature and Viscosity

Figure 5A depicts the shear stress under various conditions of curvature and height of stenosis. Curvature κ takes the values (0.00,0.5,1.0,1.5,2.0) mm^{-1} s. Thickness of stenosis ranges from 0.0 to 0.2 mm. The shear stress at $\delta = 0$ are 50.61 gram mm^{-1} s^{-2}, 55.21 gram mm^{-1} s^{-2}, 60.77 gram mm^{-1} s^{-2}, 67.48 gram mm^{-1} s^{-2}, and 75.92 gram mm^{-1}

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s^{-2} for the curvature (0.00,0.5,1.0,1.5,2.0) mm^{-1} s respectively. The values pressure 100 mm of Hg, radius 1 mm, volumetric flow rate 15 mm^3 s^{-1}, and viscosity 2.65 gram mm^{-1} s^{-1} are kept constant. When the height of stenosis is 0.05 mm, the shear stress are 60.24 gram mm^{-1} s^{-2}, 65.12 gram mm^{-1} s^{-2}, 70.86 gram mm^{-1} s^{-2}, 77.07 gram mm^{-1} s^{-2}, and 86.01 gram mm^{-1} s^{-2} for the curvature (0.00,0.5,1.0,1.5,2.0) mm^{-1} s respectively. Finally, for the stenosis thickness increases 0.2 mm, the shear stress are 202.4 gram mm^{-1} s^{-2}, 213.1 gram mm^{-1} s^{-2}, 224.9 gram mm^{-1} s^{-2}, 238.2 gram mm^{-1} s^{-2}, and 253.1 gram mm^{-1} s^{-2} for the curvature (0.00,0.5,1.0,1.5,2.0) mm^{-1} s respectively. This shows that shear stress increases with increasing curvature. It is also noted that the shear stress increases gradually with an increase in stenosis height, this figure tells us that the shear stress is affected by stenosis height and curvature.

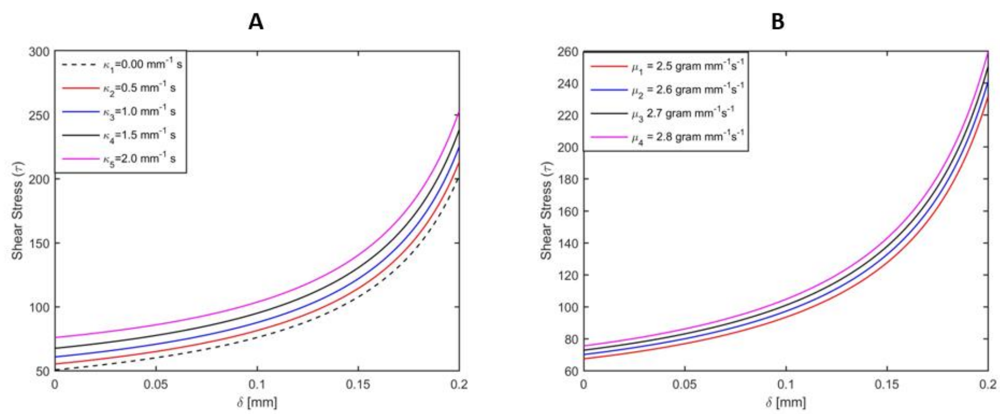

Figure 5: Shear stress A: in the case of various height of stenosis and curvature B: at the various height of stenosis and viscosity

Figure 5B depicts the relationship between the thickness of the stenosis and shear stress for different values of viscosity. Shear stress increases for increasing viscosity. At the beginning, when the thickness of the stenosis is 0.00 mm and viscosity is 2.5 gram mm^{-1} s^{-1}, the shear stress is 67.41 gram mm^{-1} s^{-2}, similarly 70.10 gram mm^{-1} s^{-2}, 72.80 gram mm^{-1} s^{-2}, and 75.50 gram mm^{-1} s^{-2} for the viscosity 2.6 gram mm^{-1} s^{-1}, 2.7 gram mm^{-1} s^{-1}, and 2.8 gram mm^{-1} s^{-1} respectively. Finally, when the stenosis thickness increases 0.2 mm, the shear stress are 231.50 gram mm^{-1} s^{-2}, 240.8 gram mm^{-1} s^{-2}, 250 gram mm^{-1} s^{-2}, and 259.30 gram mm^{-1} s^{-2} for the viscosity 2.5 gram mm^{-1} s^{-1}, 2.6 gram mm^{-1} s^{-1}, 2.7 gram mm^{-1} s^{-1}, and 2.8 gram mm^{-1} s^{-1} respectively. From these results, we conclude that the shear stress increases for increasing viscosity, when the stenosis height is kept constant, and also increases with height of the stenosis for constant curvature.

Table 2 viscosity is kept constant and curvature is increased gradually and it is shown that the shear stress is increasing for increasing height of stenosis and curvature. The curvature are increased by 0.5 mm^{-1} s in each step. For this equal increment of curvature, corresponding shear stress is shown in the table. Again for each value of curvature shear stress at 0.0 and 0.2 mm are compared. When the curvature is 0.00 mm^{-1} s the shear stress increases by 299.92%. As the curvature increases the shear stress percentage decreases gradually, which is shown in the table. The increment percentage decreases

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gradually by 14.03 %, 15.57%, %, 17.33%, 19.62% approximately. Similarly, at the lower part of the table shows relationship between increasing stenosis and shear stress for different values of viscosity. In this case the curvature to keep constant. For each value of viscosity shear stress at 0.00 and at 0.2 mm are compared. When the viscosity is 2.5 gram mm^{-1} s^{-1} the percentage increment in shear stress is 243.42% which is approximately same, as the viscosity increases by 0.1 gram mm^{-1} s^{-1}. In the above scenario, curvature exerts a more significant influence compared to viscosity on the relative importance of effects.

Table 2: Shear stress with curvature and viscosity for different height of stenosis

δ (mm)	**0.0**	**0.05**	**0.1**	**0.15**	**0.2**	**%(τ)**	**κ**
τ	50.61	60.24	76.09	107.7	202.4	299.92	0.00
τ	55.21	65.12	81.52	114.4	213.1	285.89	0.5
τ	60.73	70.86	87.78	122	224.9	270.32	1.0
τ	67.48	77.7	95.08	130.6	238.20	252.99	1.5
τ	75.92	86.01	103.07	140.5	253.10	233.37	2.0
δ (mm)	**0.00**	**0.05**	**0.1**	**0.15**	**0.2**	**% (τ)**	**μ**
τ	67.41	77.03	93.60	127.7	231.50	243.42	2.5
τ	70.10	80.11	97.34	132.8	240.8	243.50	2.6
τ	72.80	83.19	101.1	137.90	250	243.40	2.7
τ	75.50	86.27	104.80	143	259.30	243.44	2.8

4. CONCLUSION

Severe stenosis, especially in the coronary arteries supplying blood to the heart muscle, is strongly correlated with cardiovascular-related death, significantly increasing the risk of fatality. A sophisticated analysis of axisymmetric flow in a curved artery with stenosis was conducted, with the effects of curvature, viscosity, and stenosis thickness considered by incorporating a curvature term into the axial direction of the cylindrical polar form of the Navier-Stokes equations, leading to reduced partial differential equations when simplified to axisymmetric flow, with fluid dynamics in such arteries analyzed by solving these equations under specific boundary conditions. The computation of volumetric flow rate, pressure drop, shear stress, and velocity around the stenosis in a curved artery revealed significant effects of viscosity, thickness of stenosis, and curvature on the flow characteristics of blood. There is a significant effect of viscosity, thickness of stenosis and curvature on flow characteristics of the blood flow. As curvature and viscosity increase, the velocity of blood around stenosis decreases, reaching zero at the artery wall, and the velocity decreases as the artery radius increases. The volumetric flow rate at various stenosis thicknesses shows linearity, with flow decreasing as curvature and viscosity increase, particularly at a thickness of $\delta = 0.2$ mm. Furthermore, we observe that pressure drops increase with higher curvature, viscosity, and stenosis height. This indicates a uniform effect of curvature, viscosity, and stenosis thickness on pressure drop. Variations in stenosis thickness across curved arteries closely correlate with increased pressure drop, while curvature and viscosity vary. Pressure drop exhibits a slightly parabolic trend across different stenosis thicknesses, with minimal deviation at $\delta = 0$ for varying curvature and viscosity. Increasing curvature, viscosity, and stenosis thickness lead to higher shear stress. Shear stress follows a parabolic trend across various stenosis thicknesses, with minimum deviation at δ=0, regardless of curvature and viscosity. Graphical representation and analysis vividly illustrate the impact of stenosis thickness, viscosity, and curvature. Thus, we conclude that these factors significantly influence shear stress and must not be overlooked. Our mathematical model enhances the quantitative understanding of vascular disorders, potentially improving patient care and outcomes in cardiovascular medicine.

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